Coding for Channels with Rate-limited Side Information

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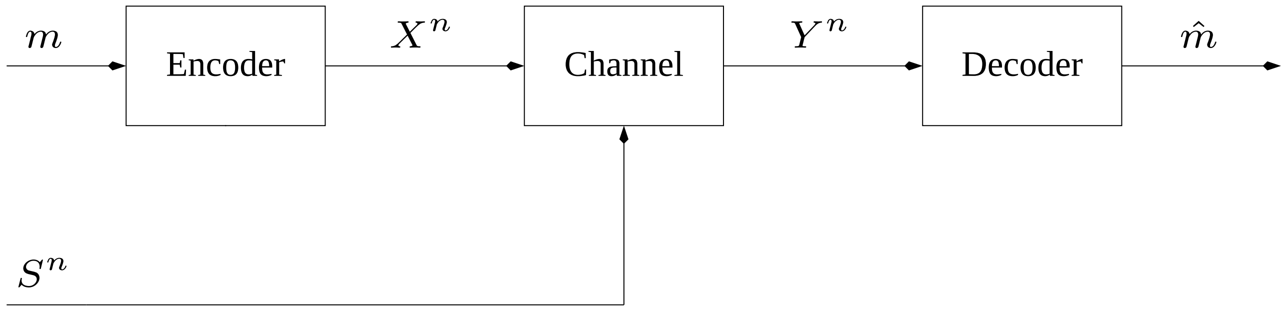
The 2006 Information Theory Workshop—ITW '06:

Punta del Este, Uruguay, March 2006

Outline

- Problem formulation
- Motivation:
  - Communication Systems
  - Watermarking
- Previous work
- Main result
- Extensions and future work

Problem Formulation



- Memoryless channel $P_{Y|X,S}(y|x,s)$ and state $P_S(s)$

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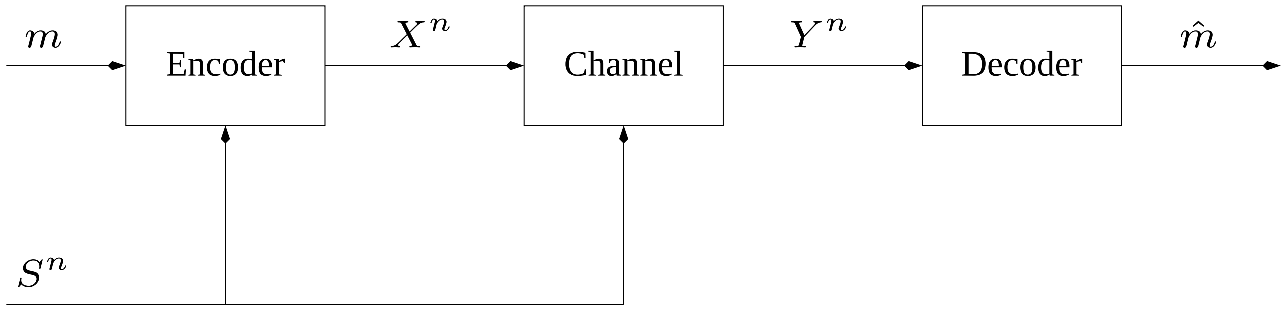

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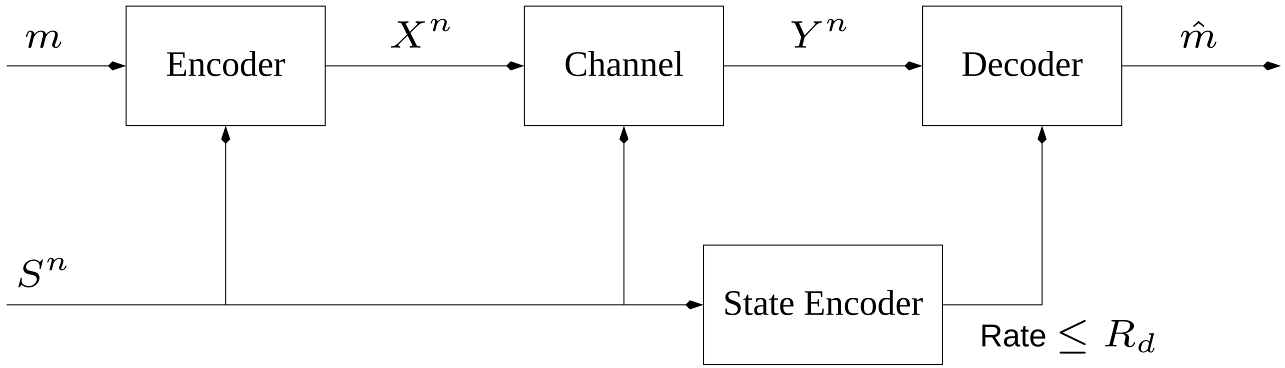

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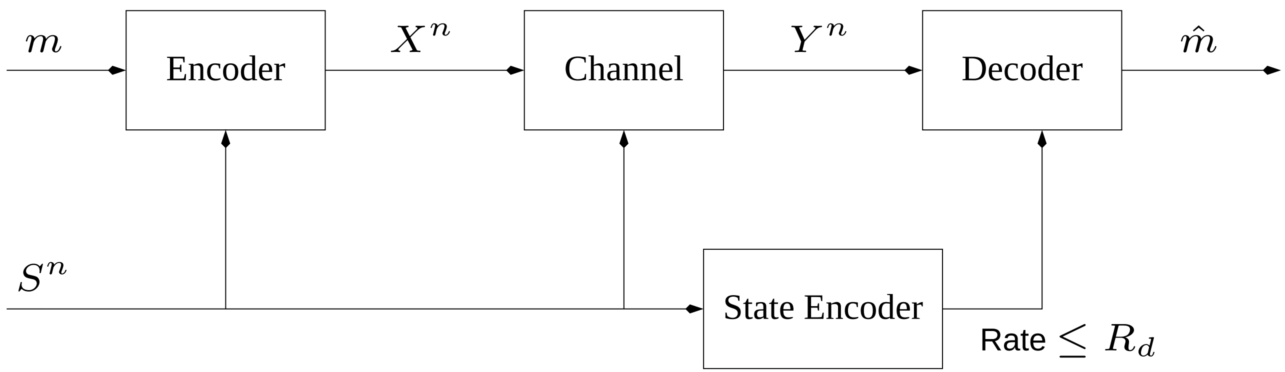

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We are interested in the region of all achievable rates and input costs:

$$R = \frac{\log|\mathcal{M}|}{n}, \quad R_d = \frac{\log|\mathcal{T}|}{n}, \quad \Gamma = E\phi(X^n).$$

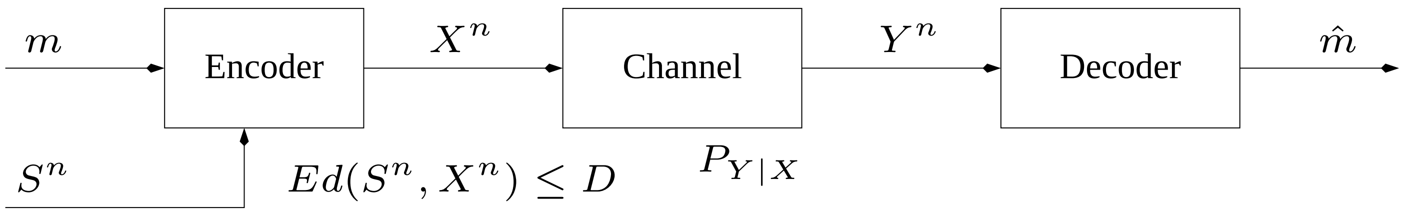
Motivation

- Communication systems:

  OFDM + coding, where coding is done across frequencies. The sender knows channels states (fading), and sends it via a wayside channel to the receiver.

- Watermarking (WM) with compressed host at the decoder.

Motivation – WM (cont'd)



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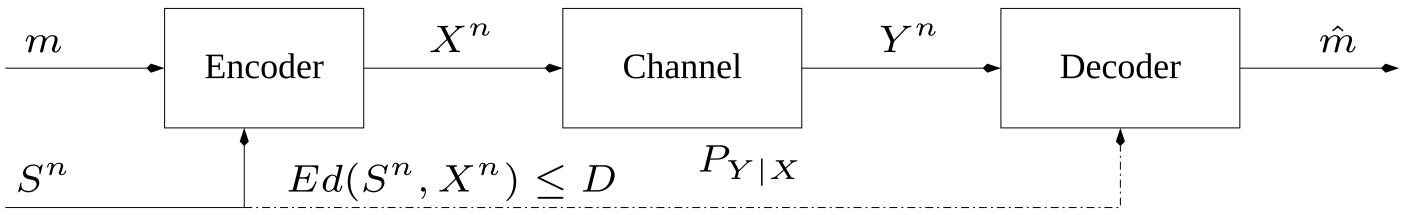

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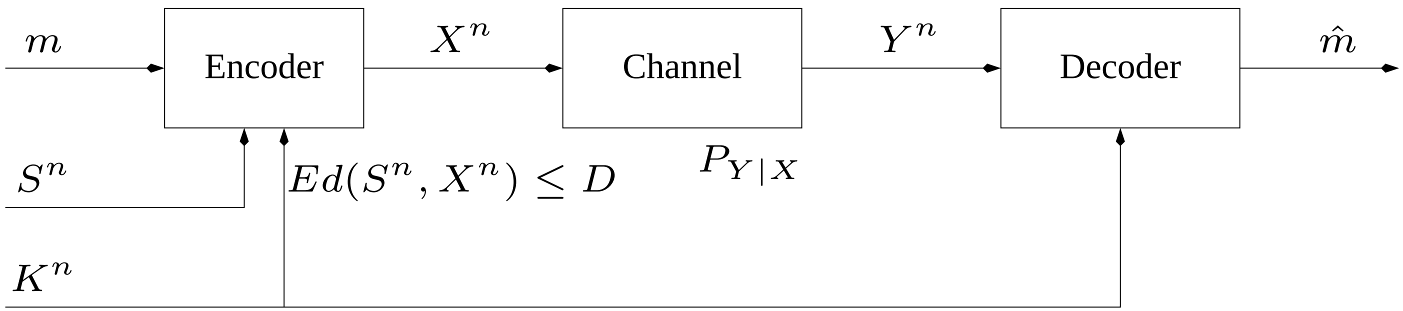

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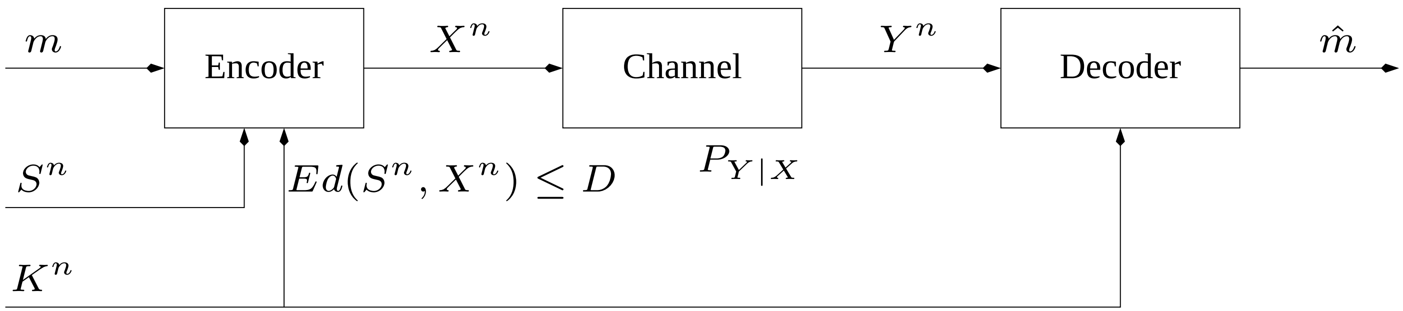

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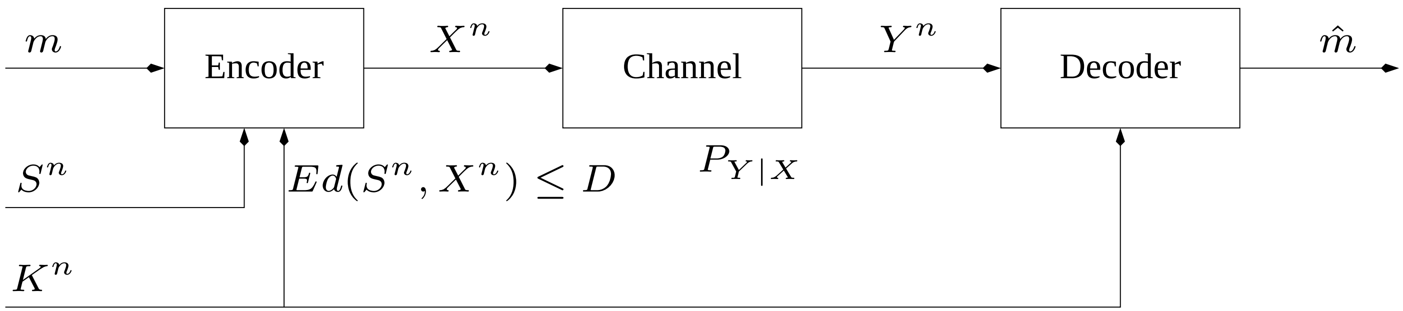

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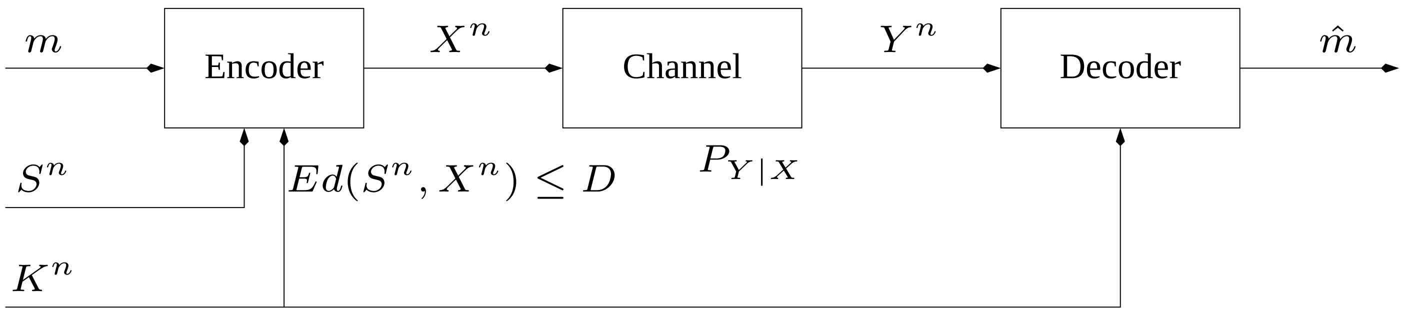

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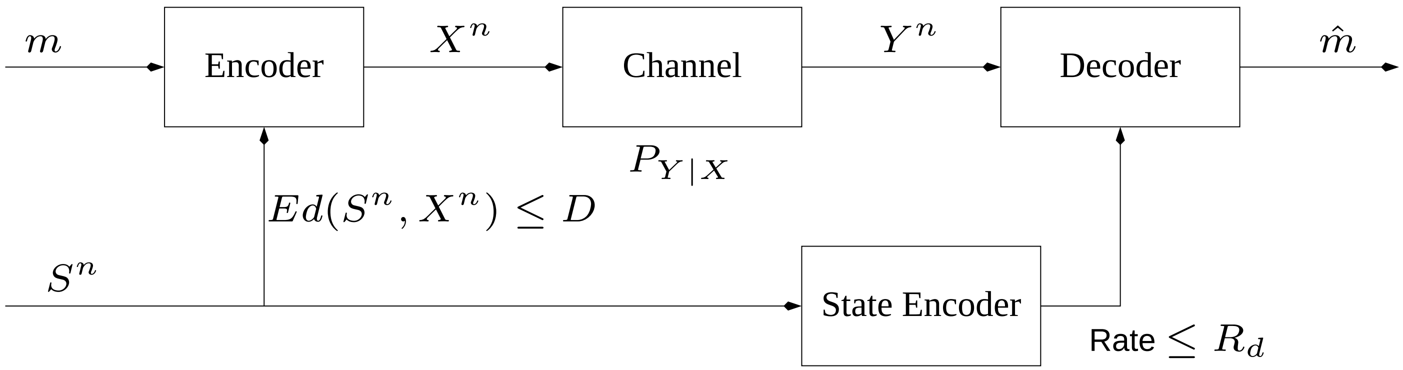
$\Rightarrow$ Quantify the deocder's a priori knowledge by a rate-limit

Motivation – WM (cont'd)



Problem:

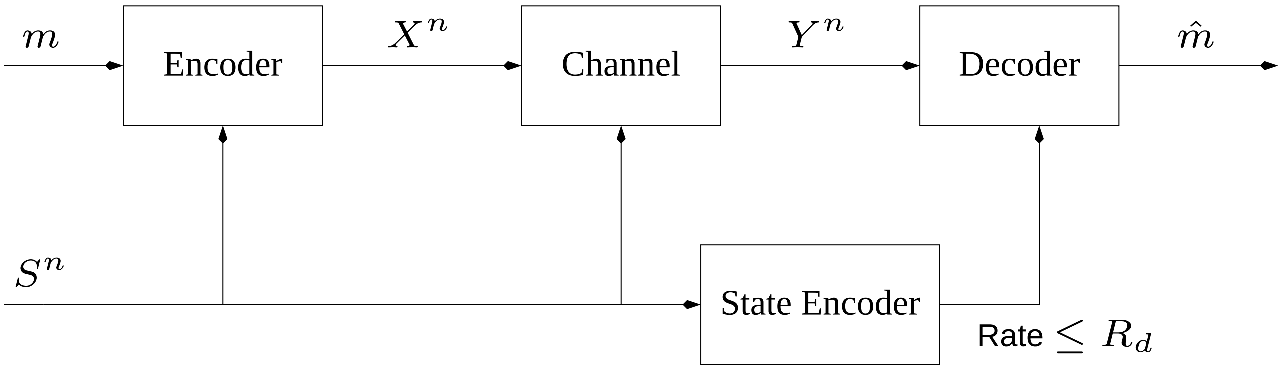
Characterize the region of all achievable (R, R_d, D), where:

R – Embedding rate,

R_d – rate of compressed SI @ decoder

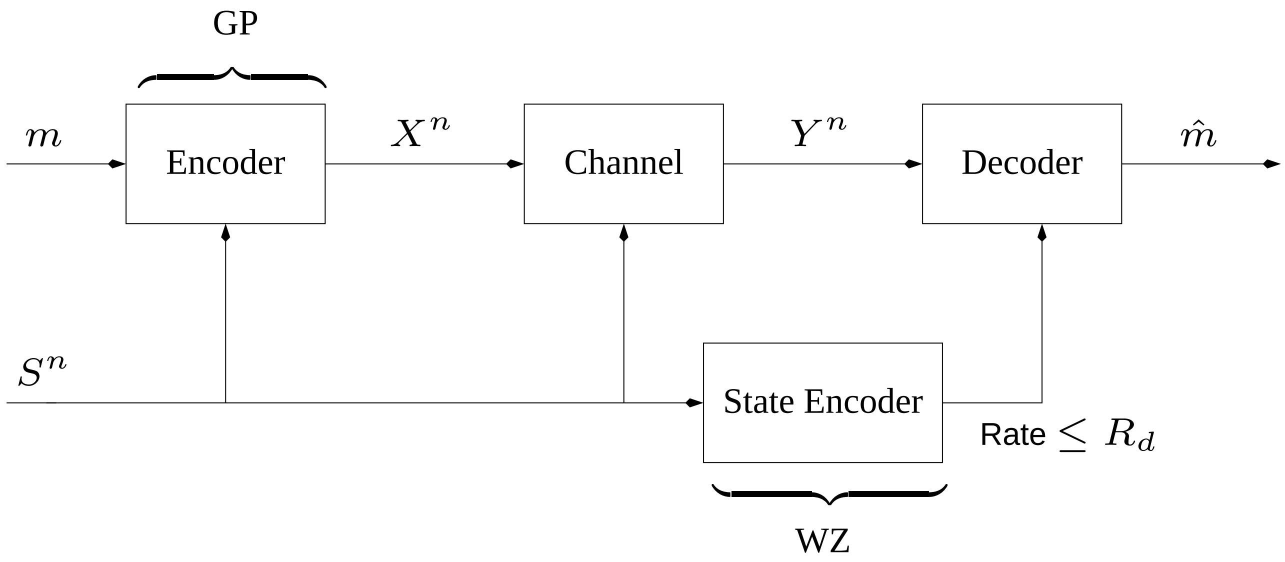
D – distortion between host and input.

Related problems



- S^n is known noncausally at the encoder $\Rightarrow$ channel coding part is related to the Gel'fand-Pinsker (GP) problem.
- Y^n depends statistically on S^n and can serve as side information (SI) in retrieving the compressed state at the decoder $\Rightarrow$ coding of S^n is related to the Wyner-Ziv (WZ) problem.

Related problems (cont'd)



- For the WZ problem, the SI Y^n is not memoryless
- There is no distortion constraint in retrieving S^n at the decoder (instead, maximize capacity of the main channel)

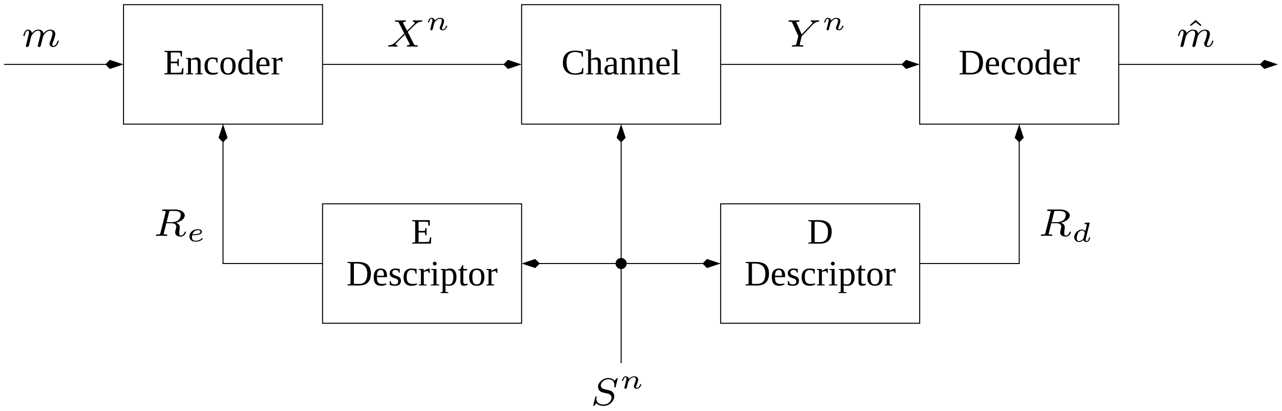
Previous work

- Wyner & Ziv, 1976
- Gel’fand & Pinsker, 1980
- Heegard & El Gamal, 1983, "On the capacity of computer memory with defects." Introduced coding for state dependent channels with rate limited side information at both ends. Devised an achievable region.

The current model is a special case of Heegard & El Gamal’s model.

Previous work (cont'd)

The Heegard & El Gamal model:



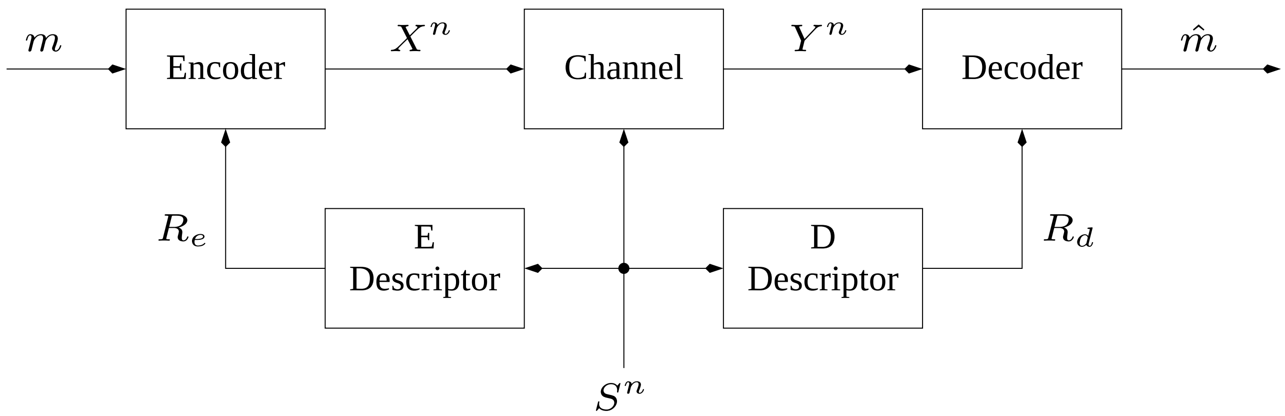
Heegard & El Gamal devised an achievable region, tight for the cases:

1. $R_e = 0$, $R_d = 0$
2. $R_e = H(S)$, $R_d = H(S|Y)$ (both sides fully informed)
3. $R_e = H(S)$, $R_d = 0$ (the GP model)
4. R_e arbitrary, $R_d = H(S|Y)$ (rate-limited SI @ encoder, fully informed decoder).

Case 4 was treated also by Rosenzweig *et al*, 2005. Dual to the problem treated here.

Previous work (cont'd)

The Heegard & El Gamal model:



Case 4. R_e arbitrary, $R_d = H(S|Y)$ (rate-limited SI @ encoder, fully informed decoder).

$$\begin{aligned} R &\leq I(X;Y|S,S_e) \\ R_e &\geq I(S:S_e) \end{aligned}$$

for some S_e such that $X \multimap S_e \multimap S \quad S_e \multimap (S,X) \multimap Y$

Previous work (cont'd)

<u>Works related to WM:</u> (very partial list)

- Moulin & O'Sullivan, 2003 – Introduced WM from IT viewpoint. Connection to GP. Bridging between public and private WM, via K^n.
- Willems & kalker, 2002 – WM system without attack channel. Two new ingerdients:
  - The host S^n is reconstructed within distortion D_2 at the decoder
  - *Composite rate limit*: a rate limit is put on the data set X^n. (Huffman code.)
- Maor & Merhav, 2005a, 2005b – Extended Willems & kalker work: (a) general lossless codes, (b) attack channel.

Main result

$\mathcal{R}^*$ – collection of all (R, R_d, Γ) satisfying

$$
\begin{aligned}
R &\leq I(U;Y|S_d) - I(U;S|S_d) \\
R_d &\geq I(S;S_d) - I(Y;S_d) \\
\Gamma &\geq \mathbb{E}\phi(X)
\end{aligned}
$$

for some (U, S_d) such that $(U, S_d) -\!\!\circ\!\!- (S, X) -\!\!\circ\!\!- Y$. Then

Theorem: For any discrete memoryless state-dependent channel, with full noncausal SI at the transmitter, and rate-limited SI at the receiver, a triple (R, R_d, Γ) is achievable if and only if $(R, R_d, \Gamma) \in \mathcal{R}^*$.

Main result (cont'd)

$\mathcal{R}^*$ – collection of all (R, R_d, Γ) satisfying

$$\begin{aligned} R &\leq I(U;Y|S_d) - I(U;S|S_d) \\ R_d &\geq I(S;S_d) - I(Y;S_d) \\ \Gamma &\geq \mathbb{E}\phi(X) \end{aligned}$$

for some (U, S_d) such that $(U, S_d) \multimap (S, X) \multimap Y$.

- S_d – A WZ rv, represents the compressed state S^n. Fully decoded, with Y^n as SI.
- U – A GP rv, represents the encoded message. Fully decoded conditioned on S_d in both sides.

Main result (cont'd)

$\mathcal{R}^*$ – collection of all (R, R_d, Γ) satisfying

$$\begin{aligned} R &\le I(U;Y|S_d) - I(U;S|S_d) \\ R_d &\ge I(S;S_d) - I(Y;S_d) \quad (*) \\ \Gamma &\ge \mathbb{E}\phi(X) \end{aligned}$$

for some (U, S_d) such that $(U, S_d) \multimap (S, X) \multimap Y$.

- $(U, S_d) \multimap (S, X) \multimap Y$ does not imply $S_d \multimap S \multimap Y$. Therefore $(*)$ is not equivalent to

  $$R_d \ge I(S; S_d|Y),$$

  full duality with GP.
- In classical WZ, $S_d \multimap S \multimap Y$ is needed to guarantee joint typicality of S_d and Y. Here it is guaranteed due to the channel.

Main result (cont'd)

$\mathcal{R}^*$ – collection of all (R, R_d, Γ) satisfying

$$\begin{aligned} R &\leq I(U;Y|S_d) - I(U;S|S_d) \\ R_d &\geq I(S;S_d) - I(Y;S_d) \\ \Gamma &\geq \mathbb{E}\phi(X) \end{aligned}$$

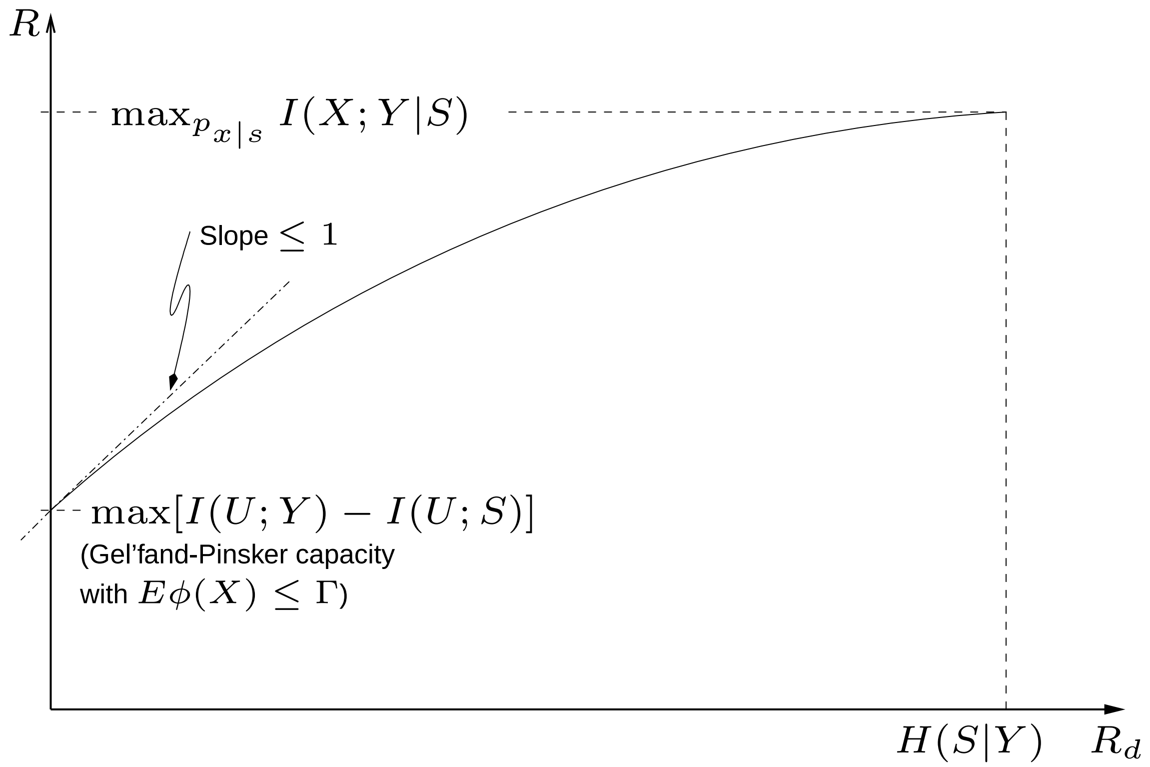
for some (U, S_d) such that $(U, S_d) \multimap (S, X) \multimap Y$.

Properties of $\mathcal{R}^*$

- $\mathcal{R}^*$ is convex
- $X = f(U, S_d, S)$, f deterministic, suffices to exhaust $\mathcal{R}^*$.

A typical (R, R_d) curve

A typical (R, R_d) curve, for fixed Γ:



- The rate allocated to provide the decoder with SI, is always at least as high as the gain in the forward rate.
- Provide SI to the decoder when the wayside channel cannot be used to transmit data – e.g.
  - Remotely located physical channel
  - WM, where a compressed host is kept in memory at the decoder, for future use.

Future work

- Extensions to networks
  - MAC, BC, etc
  - Ad hoc networks. Part of the users are silent, and can transmit SI at low cost.
- Specific models. Coding schemes.
- Computational algorithms.