The Multiple Access Channel with Two Independent States each Known Causally to One Encoder

Amos Lapidoth and Yossef Steinberg

Outline

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- Problem Formulation: The MAC with strictly causal and causal independent SI

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- Background and related results:
  - The single user channel
  - Broadcast channels
  - MAC with common SI

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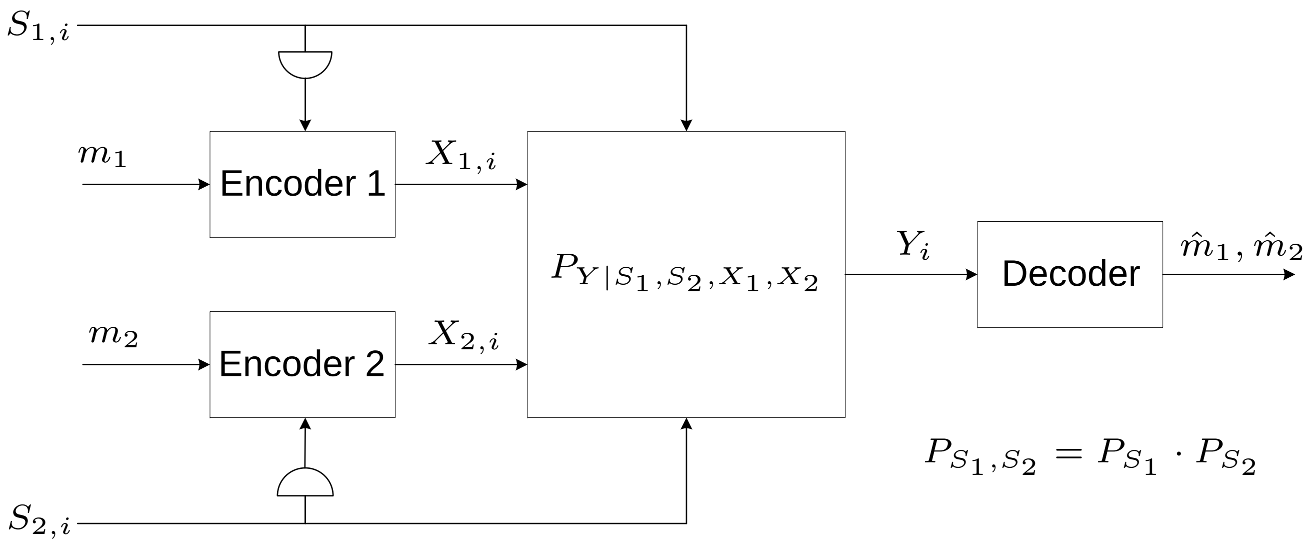
Problem Formulation

MAC with strictly causal side information (SI):



$$P_{S_1,S_2} = P_{S_1} \cdot P_{S_2}$$

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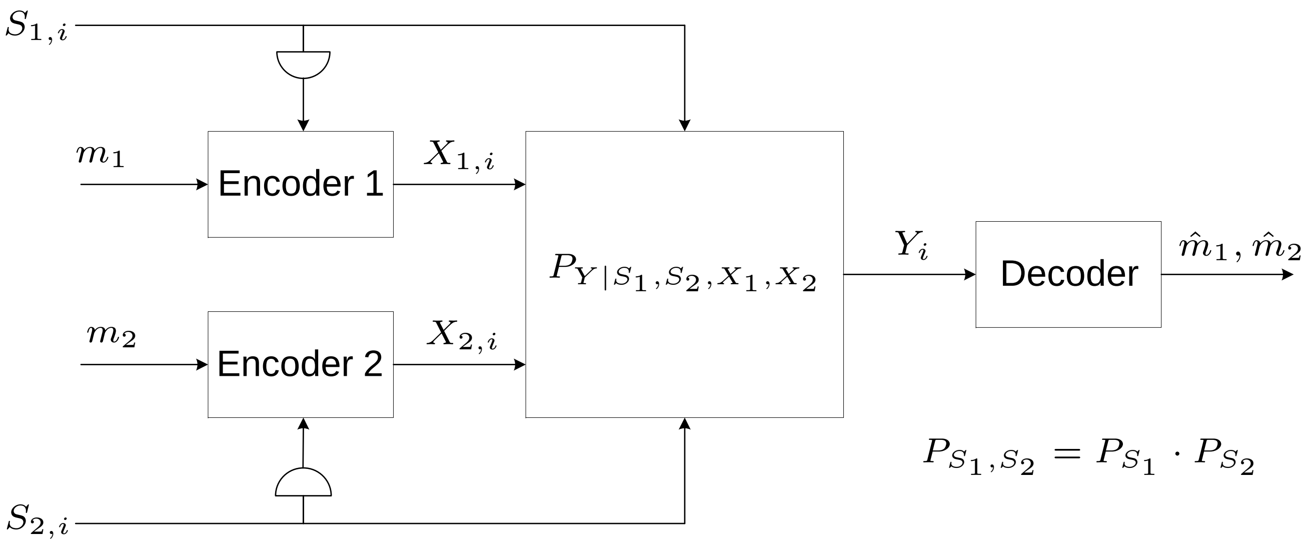

- Two independent state sequences S_1^n, S_2^n each known to one encoder in a strictly causal manner:

$$X_{1,i} = f_{1,i}(m_1, S_1^{i-1}), \quad X_{2,i} = f_{2,i}(m_2, S_2^{i-1}), \quad i = 1, \ldots, n$$

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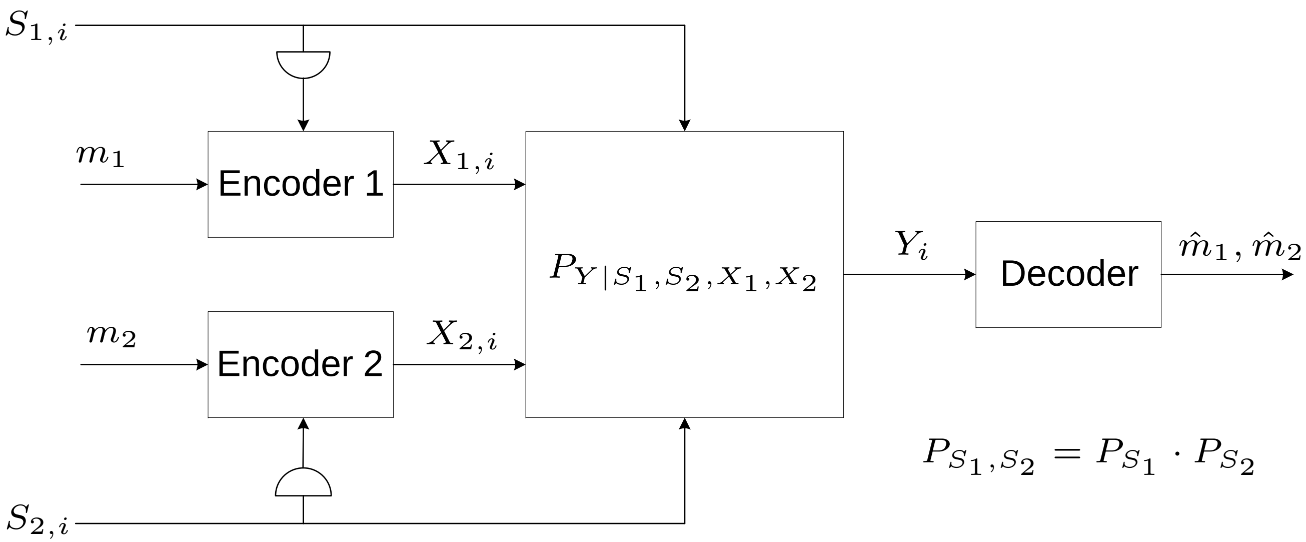

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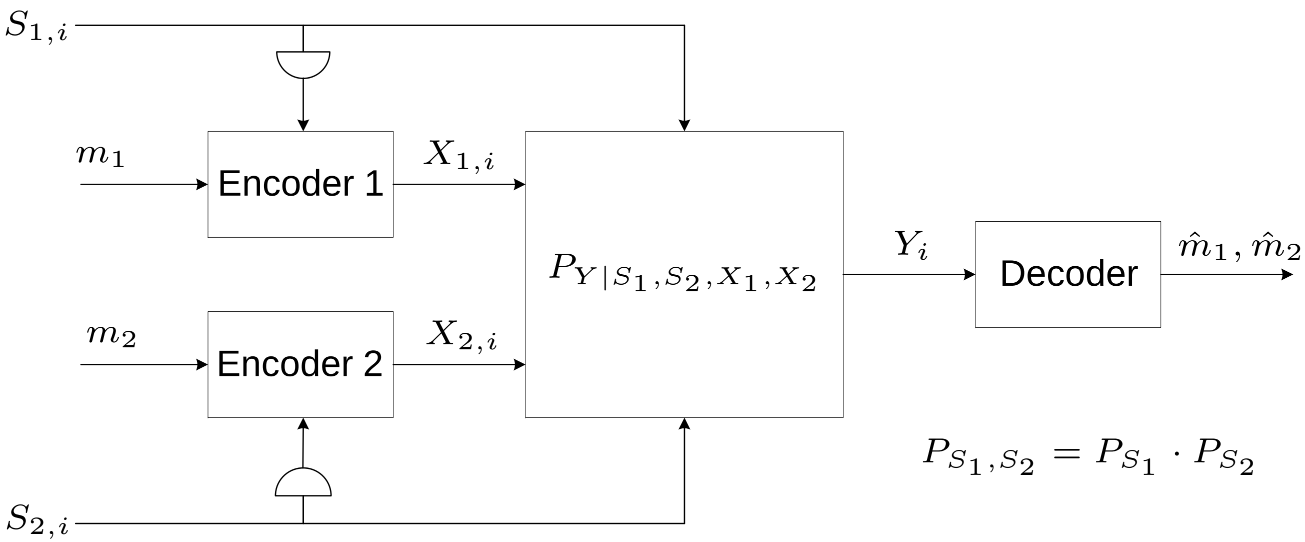

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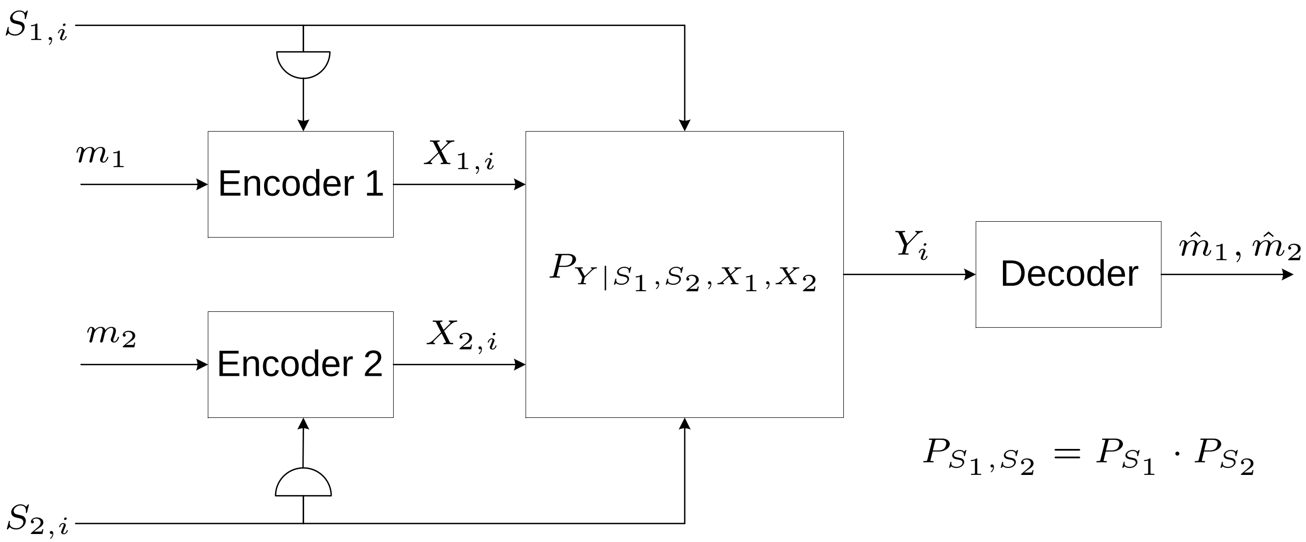

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- Memoryless, time invariant channel and states $P_{Y|S,X_1,X_2}$, P_{S_1}, P_{S_2}.

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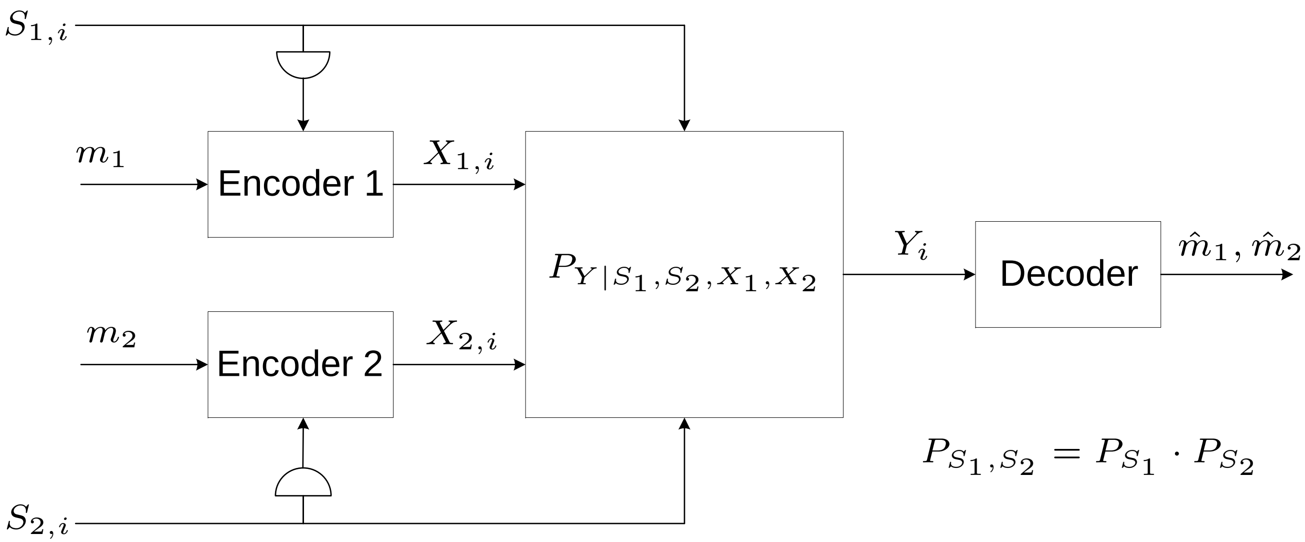

$$P_{S_1,S_2} = P_{S_1} \cdot P_{S_2}$$

We are interested in $\mathcal{C}^{\text{i}}_{\text{s-c}}$, the region of all achievable rate and cost pairs

$$(R_1, R_2, \Gamma_1, \Gamma_2).$$

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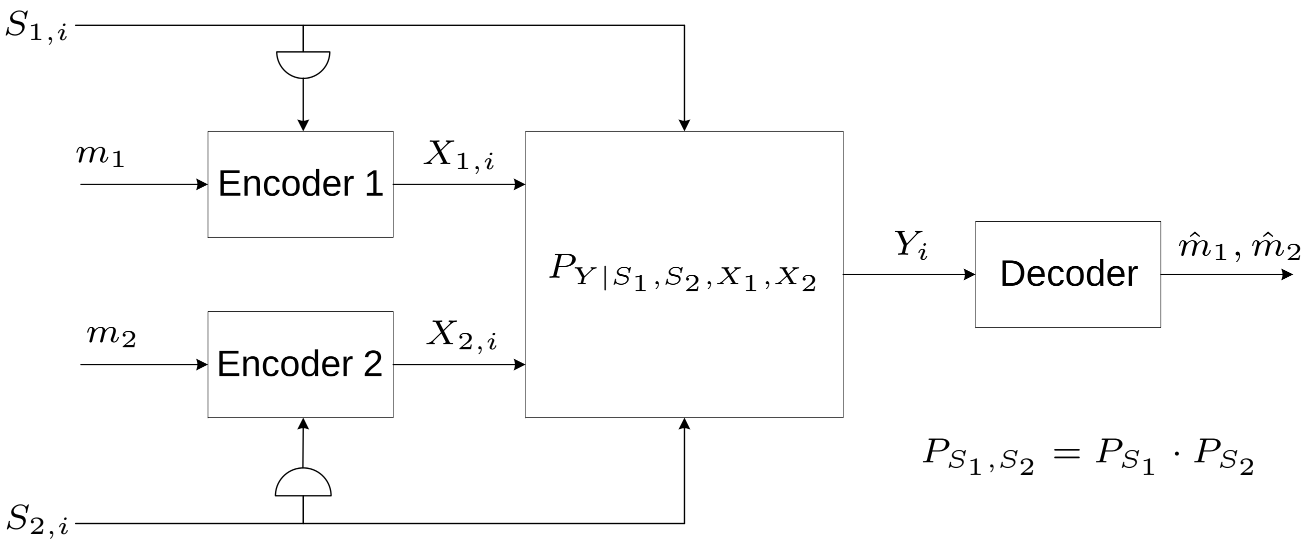

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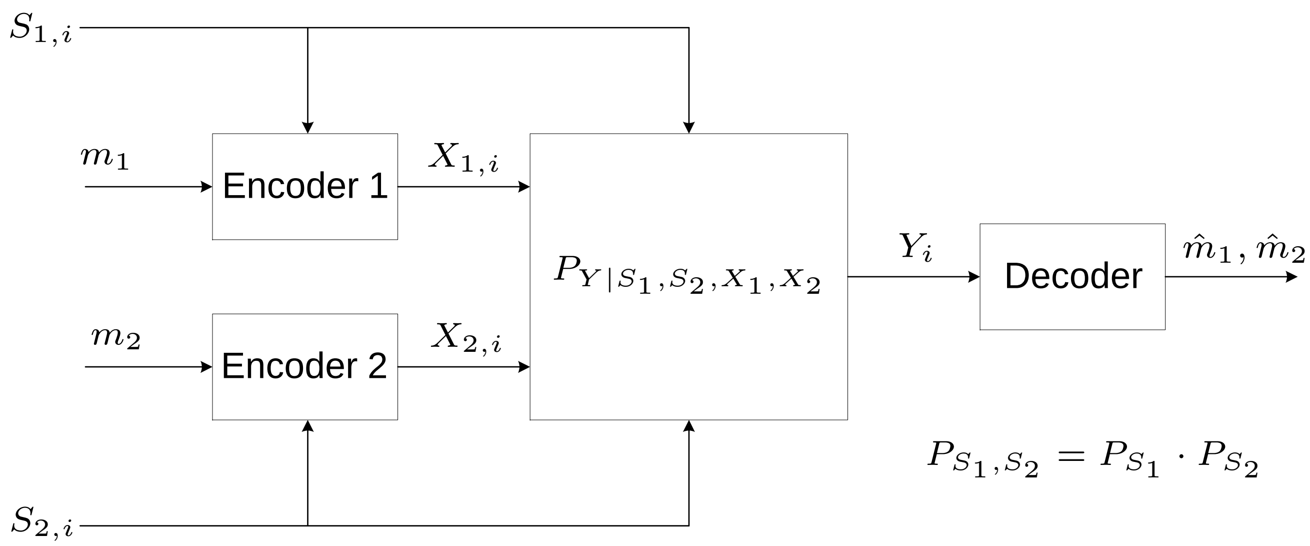
$\mathcal{C}^{\text{i}}_{\text{s-c}}(\Gamma_1, \Gamma_2)$ – the collection of all rate pairs (R_1, R_2) such that

$$(R_1, R_2, \Gamma_1, \Gamma_2) \in \mathcal{C}^{\text{i}}_{\text{s-c}}.$$

Problem Formulation

MAC with causal SI:



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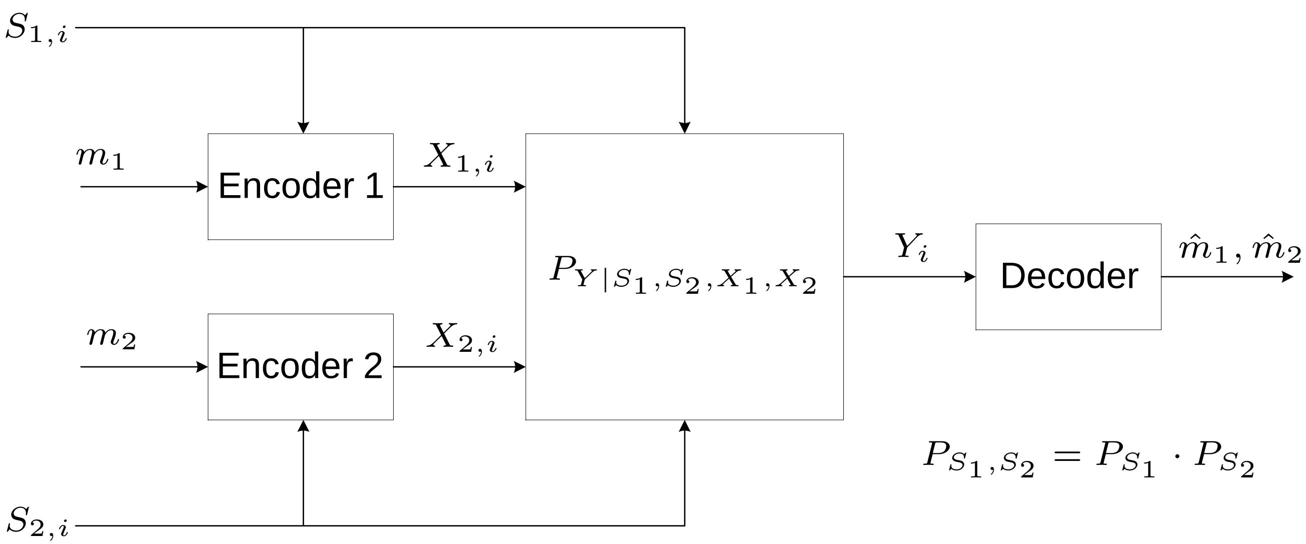

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The single user channel with SC SI

- Strictly causal SI does not increase the capacity of the single user channel

The single user channel with SC SI

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- Strictly causal SI does not increase the capacity of the single user channel

$$
\begin{aligned}
nR - n\epsilon_n &\leq I(M;Y^n) = \sum_{i=1}^{n} I(M;Y_i|Y^{i-1}) \\
&\leq \sum_{i=1}^{n} I(M,Y^{i-1};Y_i) \\
&\leq \sum_{i=1}^{n} I(M,Y^{i-1},X_i;Y_i) \\
&= \sum_{i=1}^{n} I(X_i;Y_i) \\
&\leq \max_{P_X} I(X;Y) = nC
\end{aligned}
$$

where C is the capacity without SI.

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The single user channel with SC SI

- Strictly causal SI does not increase the capacity of the single user channel (a reminiscent of the situation in feedback capacity)
- Transmission of the state (or compressed version thereof) to the other side is sub optimal: waste of precious rate, without increase in capacity.

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What about networks (BC, MAC)?

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The broadcast channel with SC SI

- An example by Dueck (1980): A non degraded additive noise BC with feedback.

  The noise is common to the two channels.

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- An example by Dueck (1980): A non degraded additive noise BC with feedback.

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  $\Rightarrow$ Equivalent to BC with strictly causal SI, where the state comprises the channel noise

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  - The encoder transmits the noise to the two users, uncompressed.

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  - Although precious rate is spent on transmitting the noise, the net effect is an increase in the capacity region.

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  - Although precious rate is spent on transmitting the noise, the net effect is an increase in the capacity region.
  - Yields gains in capacity also when only lossy transmission of the noise is possible.
- In the MAC: If the state is known to both users, they can *cooperate* in transmitting the noise (state) to the decoder. This strategy enlarges the capacity region of the MAC [Lapidoth & Steinberg, IZS 2010].

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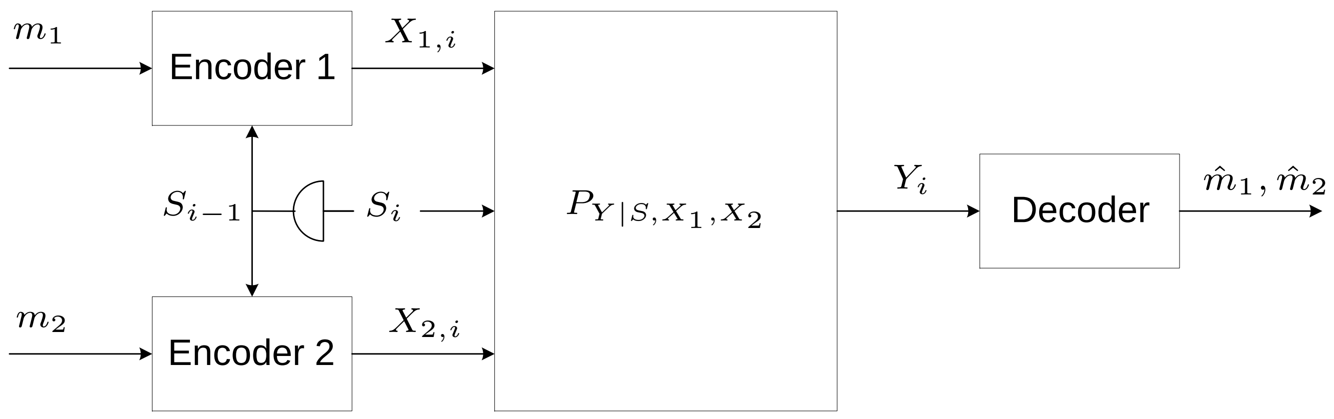
MAC with SC common SI

[Lapidoth & Steinberg, IZS2010]:



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MAC with SC common SI

$\mathcal{R}_{\text{s-c}}^{\text{common}}$ - the CH of all $(R_1, R_2, \Gamma_1, \Gamma_2)$ satisfying

$$\begin{aligned} R_1 &\leq I(X_1; Y | X_2, U, V) \\ R_2 &\leq I(X_2; Y | X_1, U, V) \\ R_1 + R_2 &\leq I(X_1, X_2; Y | U, V) \\ R_1 + R_2 &\leq I(X_1, X_2, V; Y) - I(V; S) \\ \Gamma_k &\geq \mathsf{E}[\phi_k(X_k)], \qquad k = 1, 2 \end{aligned}$$

for some joint distribution

$$P_{U,V,X_1,X_2,S,Y} = P_S P_{X_1|U} P_{X_2|U} P_U P_{V|S} P_{Y|S,X_1,X_2}.$$

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$$X_1 - U - X_2$$

$$(X_1, U, X_2) \perp (V, S)$$

MAC with SC common SI

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MAC with SC common SI

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Theorem 1 [L&S, IZS 2010]

For the MAC with strictly causal SI commonly known by the two encoders, $\mathcal{R}^{\text{common}}_{\text{s-c}}$ is achievable.

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For the MAC with strictly causal SI commonly known by the two encoders, $\mathcal{R}_{\text{s-c}}^{\text{common}}$ is achievable.

Observation: $\mathcal{R}_{\text{s-c}}^{\text{common}}$ can be strictly larger than the capacity region without SI.

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Based on MAC with common messages + block Markov scheme:

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Based on MAC with common messages + block Markov scheme:

- The state sequence S^n is compressed by a Wyner-Ziv scheme, with coding random variable V, and decoder side information Y^n.

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Based on MAC with common messages + block Markov scheme:

- The state sequence S^n is compressed by a Wyner-Ziv scheme, with coding random variable V, and decoder side information Y^n.
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Based on MAC with common messages + block Markov scheme:

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- The compressed state is transmitted to the decoder in the *next transmission block* as a *common message*, together with the independent messages m_1, m_2.
- Cooperation is possible, since the state is common.

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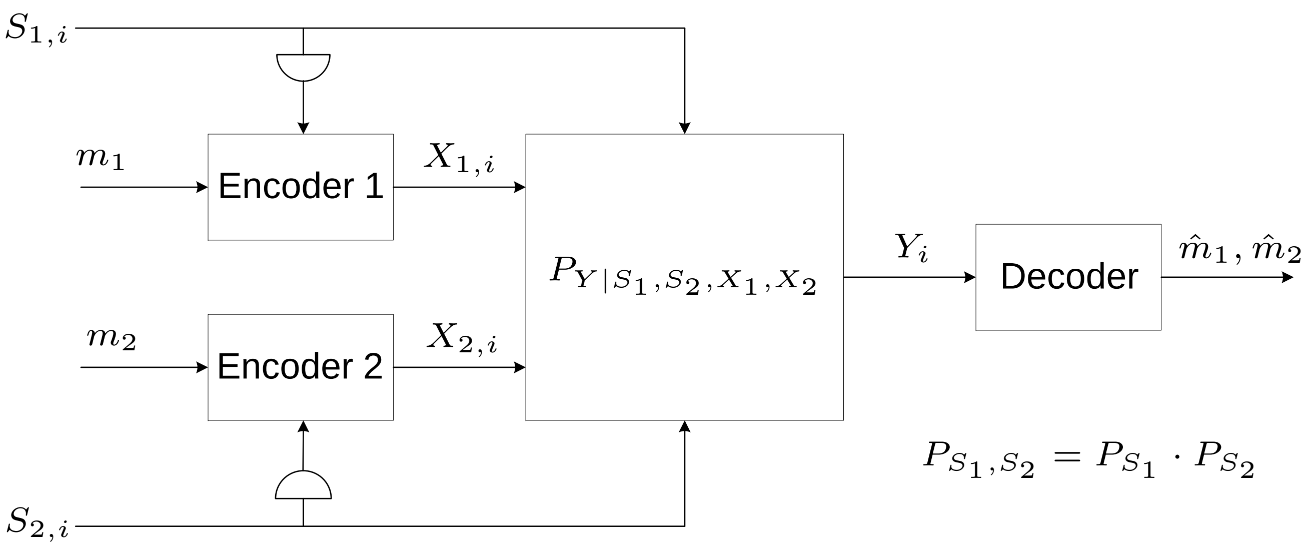

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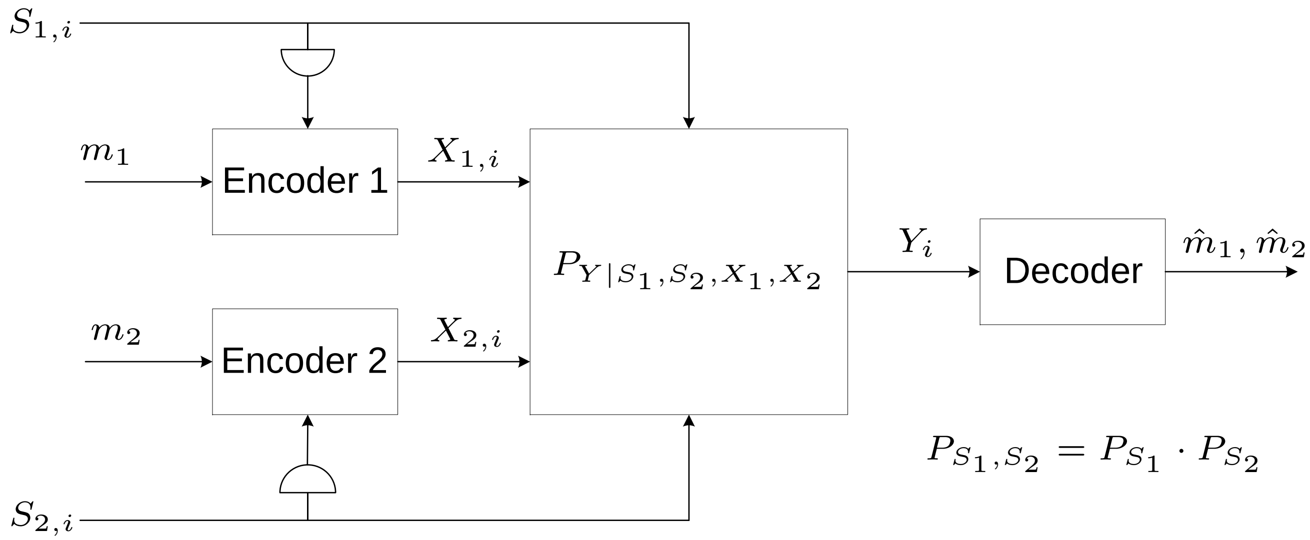

- The two encoders cannot establish cooperation of any kind

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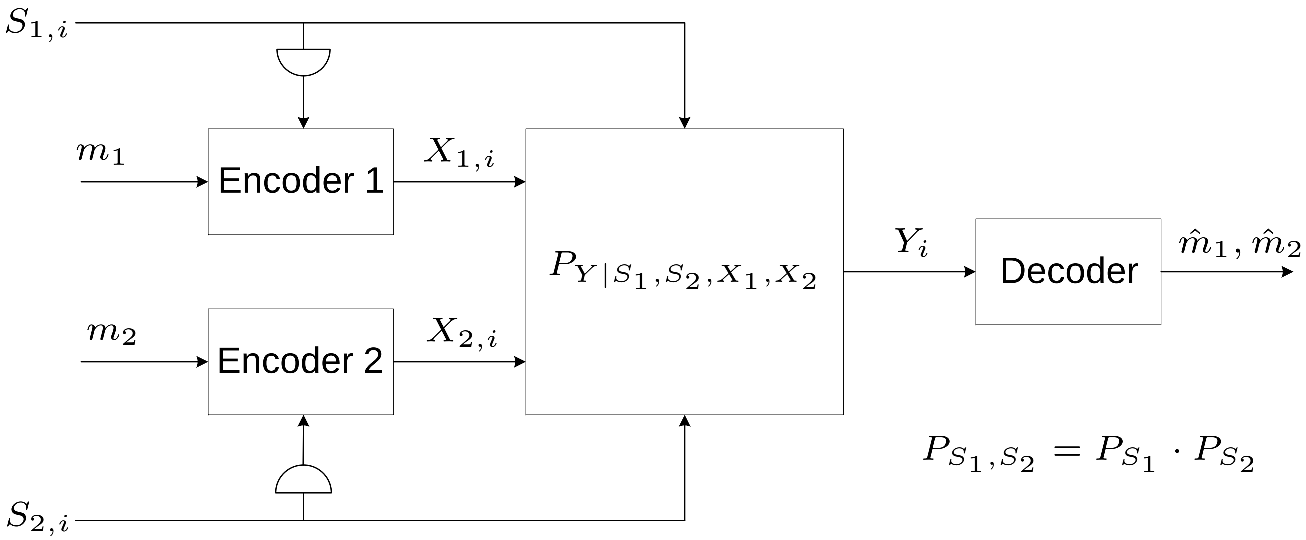

$$P_{S_1,S_2} = P_{S_1} \cdot P_{S_2}$$

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  Joint transmission of the states is not possible.

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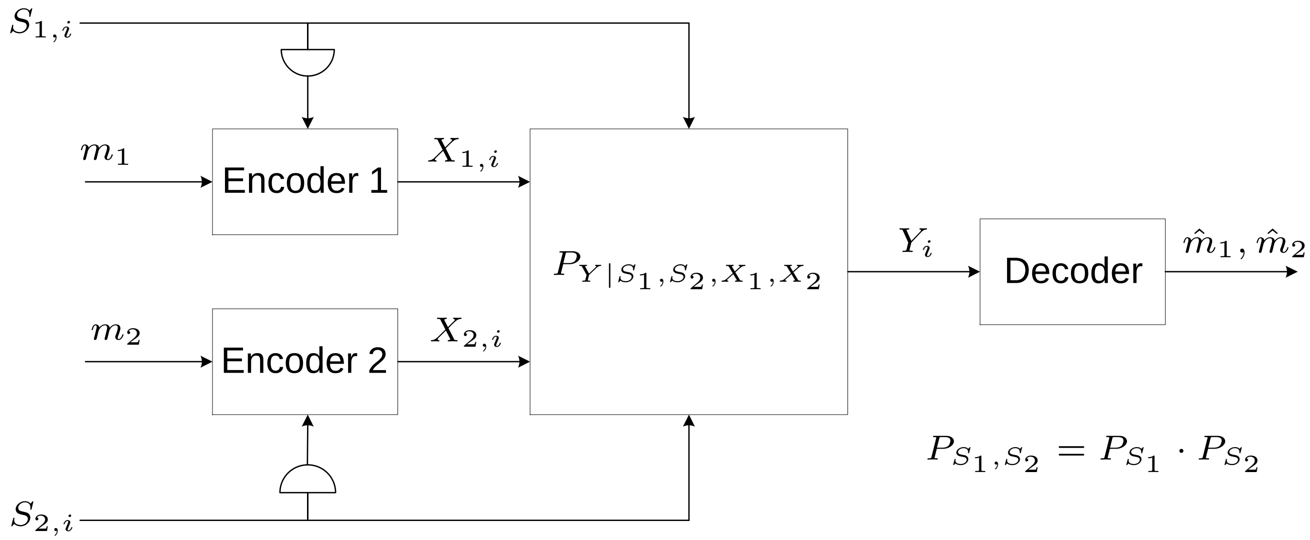

$$P_{S_1,S_2} = P_{S_1} \cdot P_{S_2}$$

- The two encoders cannot establish cooperation of any kind
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- Each of the encoders is working alone – like in the single user channel.

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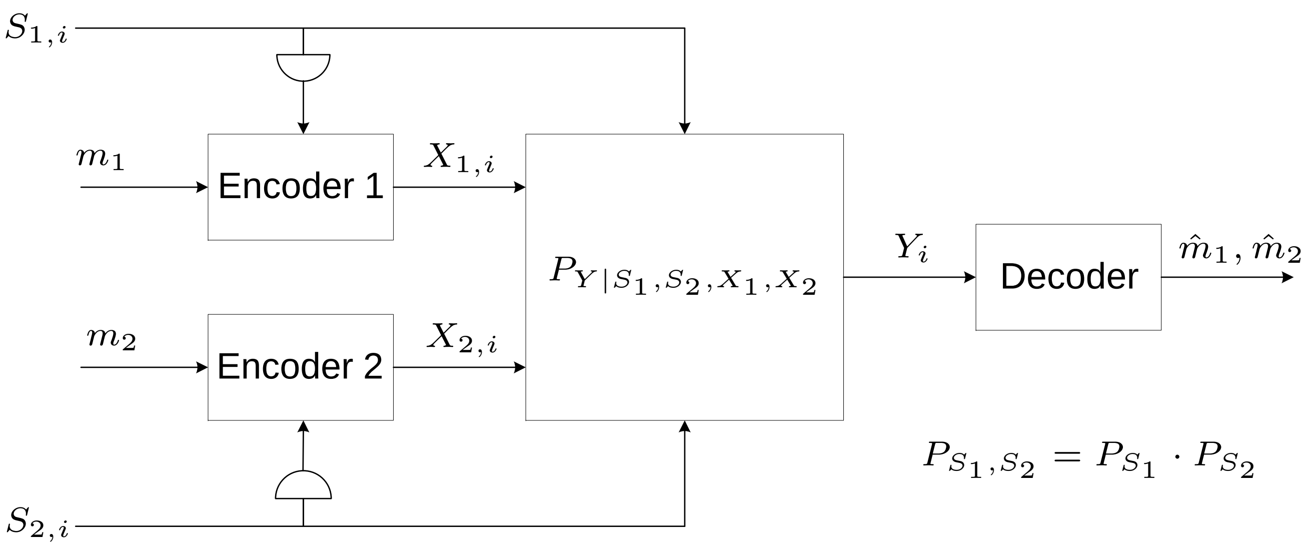

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  - In this setup, is SC SI beneficial at all?

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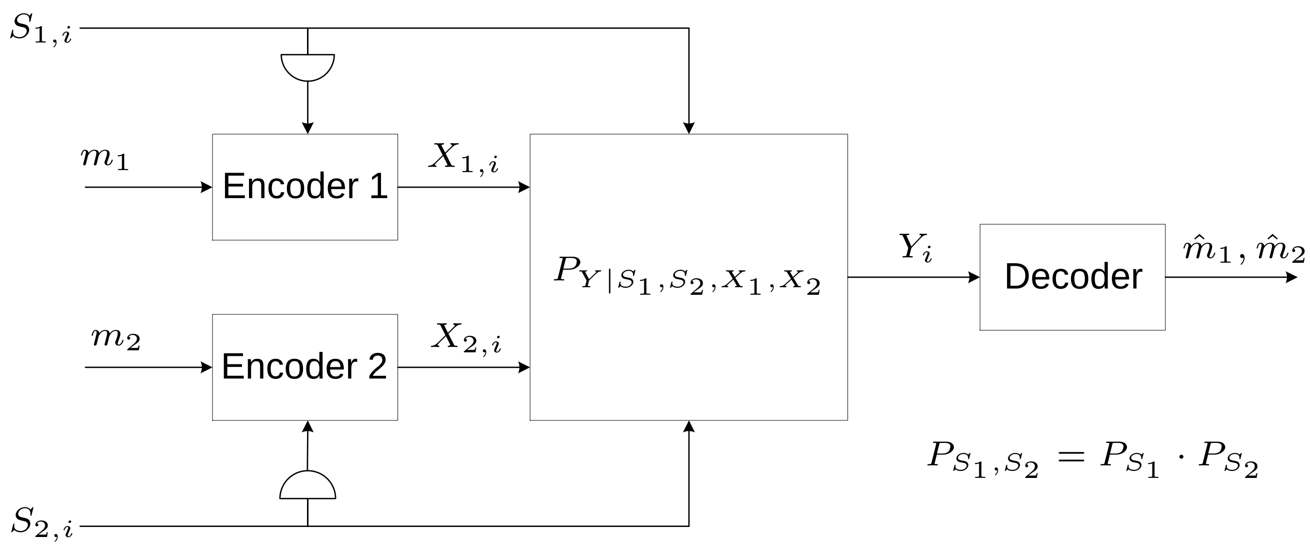

- The two encoders cannot establish cooperation of any kind

  Joint transmission of the states is not possible.
- Each of the encoders is working alone – like in the single user channel.
  - In this setup, is SC SI beneficial at all?
  - If it is beneficial, is it a good idea to compress and transmit the states to the other side?

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Let $\mathcal{R}^{\text{i}}_{\text{sc}}$ be the convex hull of the collection of all $(R_1, R_2, \Gamma_1, \Gamma_2)$ satisfying

$$\begin{aligned} 0 \le R_1 &\le I(X_1; Y|X_2, V_1, V_2) - I(V_1; S_1|Y, V_2) \\ 0 \le R_2 &\le I(X_2; Y|X_1, V_1, V_2) - I(V_2; S_2|Y, V_1) \\ R_1 + R_2 &\le I(X_1, X_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y) \\ \Gamma_k &\ge \mathbb{E}\phi_k(X_k), \qquad k = 1, 2 \end{aligned}$$

for some $(V_1, V_2, S_1, S_2, X_1, X_2, Y)$ with joint distribution

$$P_{V_1|S_1} P_{V_2|S_2} P_{S_1} P_{S_2} P_{X_1} P_{X_2} P_{Y|S_1,S_2,X_1,X_2}.$$

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$$V_1 - S_1 - (V_2, Y, S_2)$$

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$$(V_1, V_2) - (S_1, S_2) - Y$$

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X_1, X_2 are independent of each other and of the quadruple (V_1, V_2, S_1, S_2).

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X_1, X_2 are independent of each other and of the quadruple (V_1, V_2, S_1, S_2).

$$(V_1, S_1) \perp (V_2, S_2)$$

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Theorem 2 (Strictly-Causal, independent SI streams)

$$\mathcal{R}^{\mathsf{i}}_{\mathsf{sc}} \subseteq \mathcal{C}^{\mathsf{i}}_{\mathsf{sc}}$$

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\begin{aligned}
0 \leq R_1 &\leq I(X_1; Y | X_2, V_1, V_2) - I(V_1; S_1 | Y, V_2) \\
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A block Markov scheme:

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A block Markov scheme:

- The state sequences S_1^n, S_2^n are compressed by a *distributed* Wyner-Ziv scheme, with coding random variable V_1, V_2 and decoder side information Y^n.

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- The compressed states are transmitted to the decoder in the *next transmission block* as *independent codewords*, together with the independent messages m_1, m_2.

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- The two codes are decoupled.

Partial characterizations

Two propositions – about the sum rate, and about the asymmetric case.

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Partial characterizations

Two propositions – about the sum rate, and about the asymmetric case.

Proposition 1 *Strictly-causal independent SI does not increase the sum-rate capacity:*

$$\mathcal{C}^{\mathsf{i}}_{\Sigma,\text{s-c}}(\Gamma_1,\Gamma_2) = \max I(X_1, X_2; Y),$$

where the maximum is over all product distributions $P_{X_1}P_{X_2}$ *satisfying the input constraints*

$$\mathbb{E}\phi_k(X_k) \le \Gamma_k, \quad k = 1, 2.$$

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The asymmetric case:

Proposition 2 *Let S_2 be deterministic. Then the maximal rate of User 1 with strictly causal SI is equal to its single user capacity without SI*

$$\max\left\{R_1 : \ (R_1, 0) \in \mathcal{C}^{\mathsf{i}}_{\text{s-c}}(\Gamma_1, \Gamma_2)\right\} = \max I(X_1; Y | X_2),$$

where the maximum in the right hand side is over all $P_{X_1} P_{X_2}$ satisfying the input constraints

$$\mathbb{E}\phi_k(X_k) \leq \Gamma_k, \quad k = 1, 2.$$

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Example

The Gaussian MAC where the state S_1 comprises the channel noise, and S_2 is null:

$$Y = X_1 + X_2 + S_1, \qquad S_1 \sim \mathcal{N}\left(0, \sigma_{s_1}^2\right)$$

$$\mathsf{E}\left[X_1^2\right] \leq \Gamma_1, \qquad \mathsf{E}\left[X_2^2\right] \leq \Gamma_2.$$

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$\mathcal{C}^{\mathsf{i}}_{\mathsf{s\text{-}c}}(\Gamma_1, \Gamma_2)$ is the collection of all rate-pairs (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\left(1 + \frac{\Gamma_1}{\sigma_{s_1}^2}\right) \\ R_1 + R_2 &\leq \frac{1}{2}\log\left(1 + \frac{\Gamma_1 + \Gamma_2}{\sigma_{s_1}^2}\right). \end{aligned}$$

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Proof:

Direct part: good choice of random variables in $\mathcal{R}^{\mathsf{i}}_{\text{sc}}$.

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Converse: use Propositions 1 and 2.

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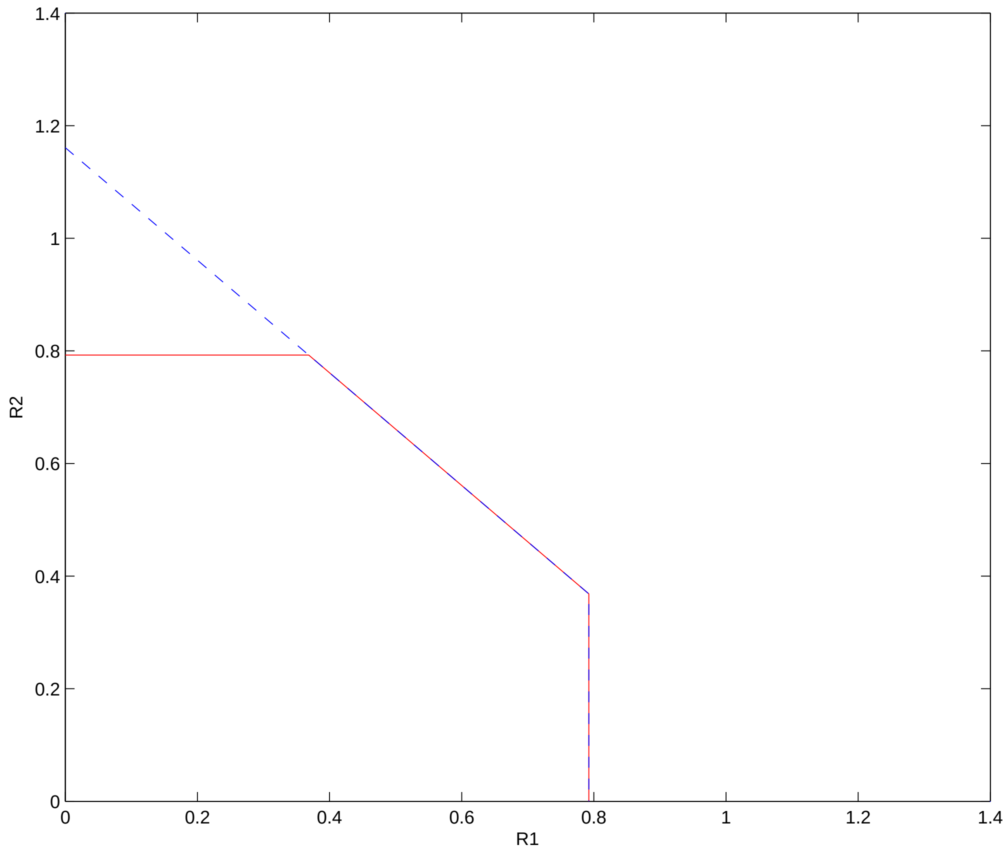

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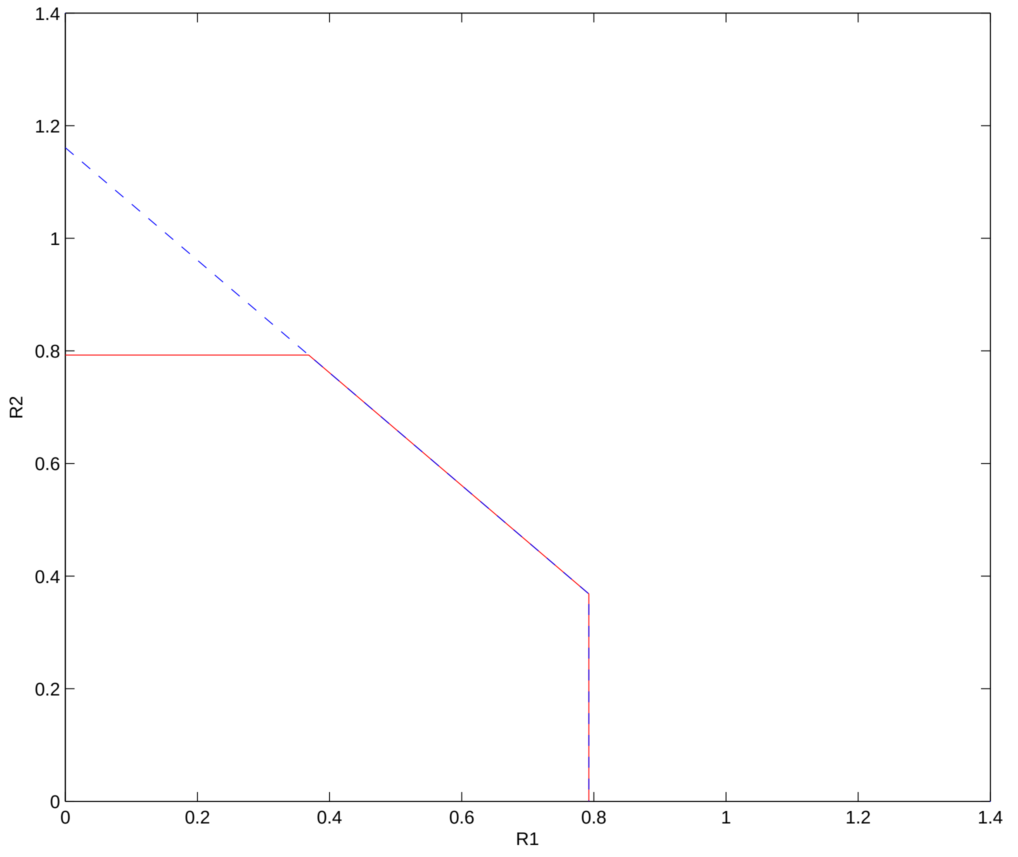

- User 1 knows the noise in a strictly causal manner, but cannot utilize it to increase his own rate.

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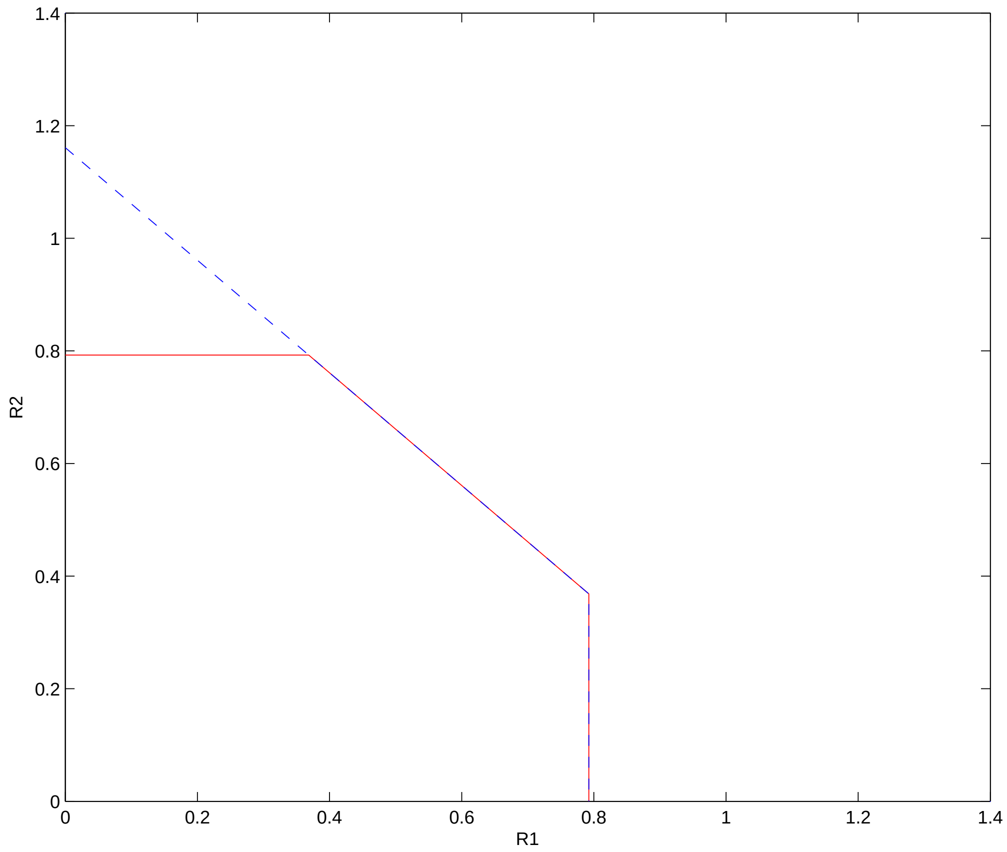

- User 1 knows the noise in a strictly causal manner, but cannot utilize it to increase his own rate.
- He can use it to increase the rate of User 2.

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MAC with causal SI

The region we had for the strictly causal case is still achievable

$$
\begin{aligned}
0 \leq R_1 &\leq I(X_1;Y|X_2,V_1,V_2) - I(V_1;S_1|Y,V_2) \\
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MAC with causal SI

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\Gamma_k &\geq \mathbb{E}\phi_k(X_k), \qquad k = 1,2
\end{aligned}
$$

with the Markov conditions

$$
\begin{gathered}
V_1 - S_1 - (V_2, Y, S_2) \\
V_2 - S_2 - (V_1, Y, S_1) \\
(V_1, V_2) - (S_1, S_2) - Y \\
X_1 \perp X_2, \qquad (X_1, X_2) \perp (V_1, V_2, S_1, S_2).
\end{gathered}
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MAC with causal SI

The region we had for the strictly causal case is still achievable

$$\begin{aligned} 0 \le R_1 &\le I(X_1;Y|X_2,V_1,V_2) - I(V_1;S_1|Y,V_2) \\ 0 \le R_2 &\le I(X_2;Y|X_1,V_1,V_2) - I(V_2;S_2|Y,V_1) \\ R_1 + R_2 &\le I(X_1,X_2;Y|V_1,V_2) - I(V_1,V_2;S_1,S_2|Y) \\ \Gamma_k &\ge \mathbb{E}\phi_k(X_k), \qquad k=1,2 \end{aligned}$$

with the Markov conditions

$$\begin{gathered} V_1 - S_1 - (V_2, Y, S_2) \\ V_2 - S_2 - (V_1, Y, S_1) \\ (V_1, V_2) - (S_1, S_2) - Y \\ X_1 \perp X_2, \qquad (X_1, X_2) \perp (V_1, V_2, S_1, S_2). \end{gathered}$$

But now, X_1, X_2 can depend on S.

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But now, X_1, X_2 can depend on S.

$\Rightarrow$ Use Shannon strategies on top of our block Markov scheme.

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$$\begin{aligned} 0 \le R_1 &\le I(X_1;Y|X_2,V_1,V_2) - I(V_1;S_1|Y,V_2) \\ 0 \le R_2 &\le I(X_2;Y|X_1,V_1,V_2) - I(V_2;S_2|Y,V_1) \\ R_1 + R_2 &\le I(X_1,X_2;Y|V_1,V_2) - I(V_1,V_2;S_1,S_2|Y) \\ \Gamma_k &\ge \mathbb{E}\phi_k(X_k), \qquad k=1,2 \end{aligned}$$

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But now, X_1, X_2 can depend on S.

Replace (X_1, X_2) by (U_1, U_2) independent of (S_1, S_2), and let

$$P_{X_1|U_1,S_1}, \quad P_{X_2|U_2,S_2}$$

Main result

$\mathcal{R}_{\text{cau}}^{\text{i}}$ - the CH of all $(R_1, R_2, \Gamma_1, \Gamma_2)$ satisfying

$$
\begin{aligned}
0 \le R_1 &\le I(U_1; Y|U_2, V_1, V_2) - I(V_1; S_1|Y, V_2) \\
0 \le R_2 &\le I(U_2; Y|U_1, V_1, V_2) - I(V_2; S_2|Y, V_1) \\
R_1 + R_2 &\le I(U_1, U_2; Y|V_1, V_2) - I(V_1, V_2; S_1, S_2|Y) \\
\Gamma_k &\ge \mathbb{E}\phi_k(X_k), \qquad k = 1, 2
\end{aligned}
$$

for some $(V_1, V_2, U_1, U_2, S_1, S_2, X_1, X_2, Y)$ with joint distribution

$$P_{V_1|S_1} P_{V_2|S_2} P_{U_1} P_{U_2} P_{S_1} P_{S_2} P_{X_1|U_1,S_1} P_{X_2|U_2,S_2} P_{Y|S_1,S_2,X_1,X_2}.$$

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Main result

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$$P_{V_1|S_1} P_{V_2|S_2} P_{U_1} P_{U_2} P_{S_1} P_{S_2} P_{X_1|U_1,S_1} P_{X_2|U_2,S_2} P_{Y|S_1,S_2,X_1,X_2}.$$

Theorem 3 (Causal, independent SI streams)

$$\mathcal{R}^{\text{i}}_{\text{cau}} \subseteq \mathcal{C}^{\text{i}}_{\text{cau}}$$

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The naïve approach

The naïve approach – using Shannon strategies, without block Markov coding of the state.

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The naïve approach

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all (R_1, R_2) satisfying

$$R_1 \leq I(T_1; Y | T_2, Q)$$

$$R_2 \leq I(T_2; Y | T_1, Q)$$

$$R_1 + R_2 \leq I(T_1, T_2; Y | Q)$$

for some joint distribution $P_Q P_{T_1|Q} P_{T_2|Q} P_{Y|T_1,T_2}$.

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for some joint distribution $P_Q P_{T_1|Q} P_{T_2|Q} P_{Y|T_1,T_2}$. Here

$T_k,\, k = 1, 2$ are random Shannon strategies:

$$T_k \in \mathcal{T}_k, \quad \text{the set of mappings} \quad t_k : \mathcal{S}_k \to \mathcal{X}_k$$

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$$T_k \in \mathcal{T}_k, \quad \text{the set of mappings} \quad t_k : \mathcal{S}_k \to \mathcal{X}_k$$

Q is a time sharing random variable,

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$$
\begin{aligned}
R_1 &\le I(T_1; Y|T_2, Q) \\
R_2 &\le I(T_2; Y|T_1, Q) \\
R_1 + R_2 &\le I(T_1, T_2; Y|Q)
\end{aligned}
$$

for some joint distribution $P_Q P_{T_1|Q} P_{T_2|Q} P_{Y|T_1,T_2}$. Here

T_k, $k = 1, 2$ are random Shannon strategies:

$T_k \in \mathcal{T}_k$, the set of mappings $t_k : \mathcal{S}_k \to \mathcal{X}_k$

Q is a time sharing random variable, and

$$
\begin{aligned}
P_{Y|T_1,T_2}(y|t_1, t_2) &= \sum_{s_1 \in \mathcal{S}_1} \sum_{s_2 \in \mathcal{S}_2} P_{S_1}(s_1) P_{S_2}(s_2) \\
&\quad \cdot P_{Y|S_1,S_2,X_1,X_2}\big(y|s_1, s_2, t_1(s_1), t_2(s_2)\big).
\end{aligned}
$$

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\end{aligned}
$$

for some joint distribution $P_Q P_{T_1|Q} P_{T_2|Q} P_{Y|T_1,T_2}$.

We denote this region as $\mathcal{R}^{\text{naïve}}$.

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The naïve approach

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\leq I(T_1; Y|T_2, Q) \\ R_2 &\leq I(T_2; Y|T_1, Q) \\ R_1 + R_2 &\leq I(T_1, T_2; Y|Q) \end{aligned}$$

for some joint distribution $P_Q P_{T_1|Q} P_{T_2|Q} P_{Y|T_1,T_2}$.

We denote this region as $\mathcal{R}^{\text{naïve}}$.

$\mathcal{R}^{\text{naïve}}$ contains the region suggested in [S.A. Jafar, Dec 2006].

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The naïve approach

- $\mathcal{R}^{\text{i}}_{\text{cau}}$ contains the region of the naïve approach, since we can always choose deterministic (V_1, V_2).

- In some cases, the inclusion is strict.

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Example

The asymmetric state-dependent MAC consisting of two single user channels:

$$\mathcal{X}_1 = \{0,1\}, \quad \mathcal{X}_2 = \{0,1,2,3\}, \quad \mathcal{Y} = \mathcal{Y}_1 \times \mathcal{Y}_2$$

$$\mathcal{Y}_1 = \{0,1\}, \quad \mathcal{Y}_2 = \{0,1,2,3\}.$$

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The channel is defined as

$$\begin{aligned} Y_1 &= X_1 \\ Y_2 &= X_2 \oplus S_1, \end{aligned}$$

Example

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The channel is defined as

$$\begin{aligned} Y_1 &= X_1 \\ Y_2 &= X_2 \oplus S_1, \end{aligned}$$

where

$$\mathcal{S}_1 = \{0,1,2,3\}, \quad P_{S_1} = (1-p, p/3, p/3, p/3), \quad H(S_1) < 1.$$

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$$
\begin{aligned}
Y_1 &= X_1, && \text{binary} \\
Y_2 &= X_2 \oplus S_1, && \text{quaternary with } H(S_1) < 1.
\end{aligned}
$$

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$$Y_1 = X_1, \qquad \text{binary}$$

$$Y_2 = X_2 \oplus S_1, \qquad \text{quaternary with } H(S_1) < 1.$$

What is the maximal transmission rate of user 2 under each of the schemes?

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What is the maximal transmission rate of user 2 under each of the schemes?

- The block Markov coding scheme yields $R_{2,\text{max}}^{\text{(bm)}} = 2$.

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What is the maximal transmission rate of user 2 under each of the schemes?

- The block Markov coding scheme yields $R^{\text{(bm)}}_{2,\text{max}} = 2$.

  Achievability - by proper choice of random variables in $\mathcal{R}^{\text{i}}_{\text{cau}}$.

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  This is tight, since $|\mathcal{X}_2| = 4$.

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What is the maximal transmission rate of user 2 under each of the schemes?

- The block Markov coding scheme yields $R_{2,\text{max}}^{(\text{bm})} = 2$.

  Achievability - by proper choice of random variables in $\mathcal{R}_{\text{cau}}^{\text{i}}$.

  This is tight, since $|\mathcal{X}_2| = 4$.

- It can be shown that $R_{2,\text{max}}^{(\text{naïve})} < 2$.

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Summary

- Derived achievable region for the MAC with two independent strictly causal SI streams, based on block Markov encoding of the state.
- Although cooperation between the users is impossible in this setup, strictly causal SI enlarges the capacity region of the MAC.
- Extended the results to causal SI
- The new region for causal SI is strictly better than the region obtained by the naïve approach, which utilizes only Shannon strategies without block-Markov coding.

Thank You!