# The Multiple Access Channel with Causal and Strictly Causal Side Information at the Encoders

Amos Lapidoth and Yossef Steinberg

Problem Formulation: The MAC with strictly causal and causal common SI

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- An achievable region for the strictly causal model

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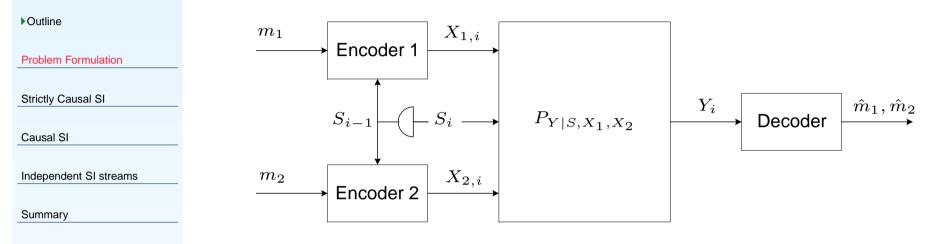
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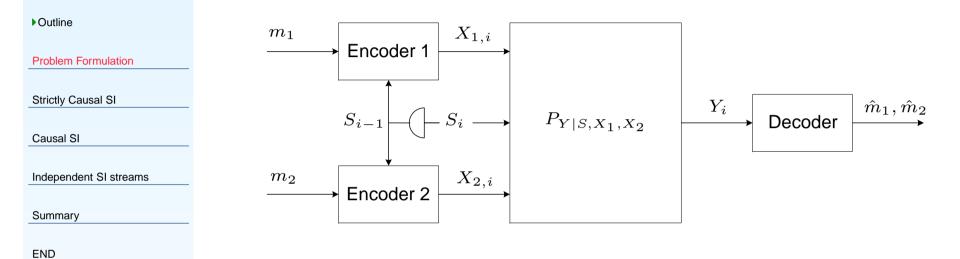
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MAC with strictly causal side information (SI):



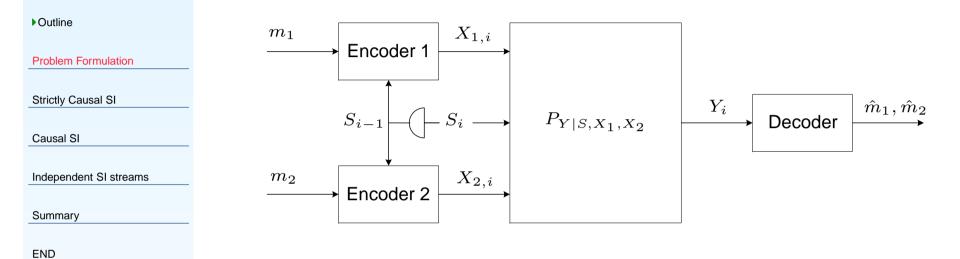
MAC with strictly causal side information (SI):



• One state sequence  $S^n$ , available to the encoders in a strictly causal manner:

$$X_{1,i} = f_{1,i}(m_1, S^{i-1}), \quad X_{2,i} = f_{2,i}(m_2, S^{i-1}), \quad i = 1, \dots, n$$

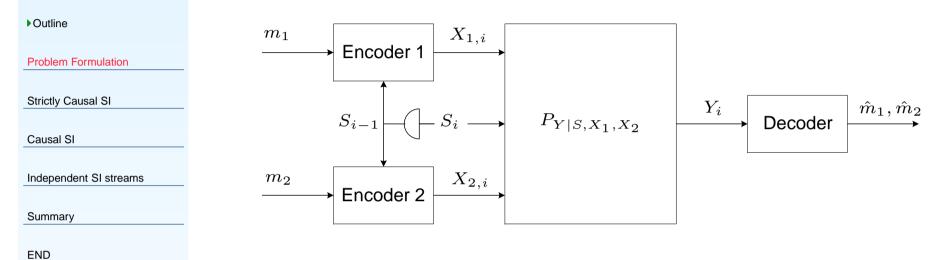
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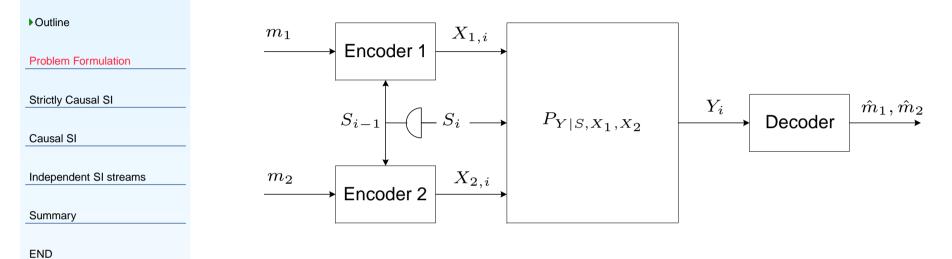


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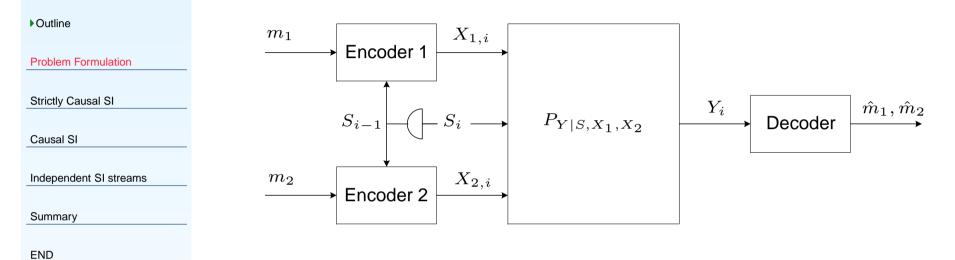
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• Memoryless, time invariant channel and state  $P_{Y|S,X_1,X_2}$ ,  $P_S$ .

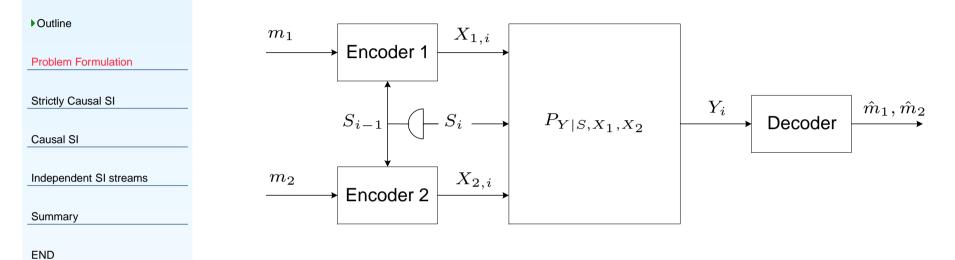
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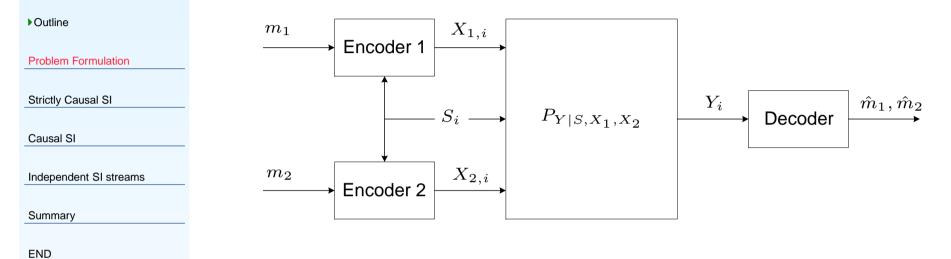
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 $C_{s-c}(\Gamma_1,\Gamma_2)$  – the collection of all rate pairs  $(R_1,R_2)$  such that

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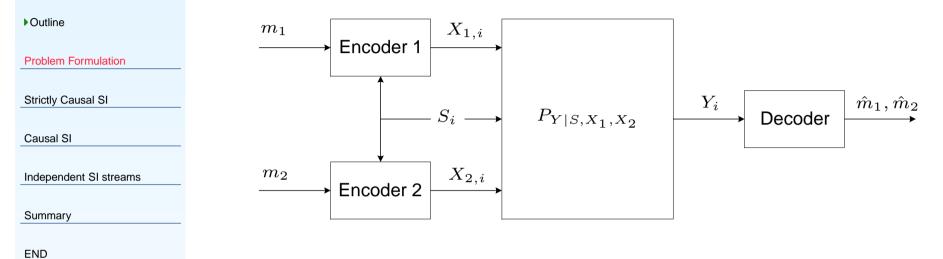
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We are interested in  $C_{cau}$ , the region of all achievable rate and cost pairs

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- Strictly causal SI does not increase the capacity of the single user channel

Outline

**Problem Formulation** 

Strictly Causal SI Single user and BC with SC SI MAC with SC SI - main result The coding scheme

Example

Causal SI

Independent SI streams

Summary

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Outline

Problem Formulation	
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Independent SI streams	
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END	

$$nR - n\epsilon_n \leq I(M; Y^n) = \sum_{i=1}^n I(M; Y_i | Y^{i-1})$$
$$\leq \sum_{i=1}^n I(M, Y^{i-1}; Y_i)$$
$$\leq \sum_{i=1}^n I(M, Y^{i-1}, X_i; Y_i)$$

$$= \sum_{i=1}^{n} I(X_i; Y_i)$$

$$\leq \max_{P_X} I(X;Y) = nC$$

where C is the capacity without SI.

- Strictly causal SI does not increase the capacity of the single user channel (a reminiscent of the situation in feedback capacity)

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- Strictly causal SI does not increase the capacity of the single user channel (a reminiscent of the situation in feedback capacity)

- Transmission of the state (or compressed version thereof) to the other side is sub optimal: waste of precious rate, without increase in capacity.

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An example by Dueck (1980): A non degraded additive noise BC with feedback.
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Knowledge of the additive noise at the decoder facilitates decoding of the messages.

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- Yields gains in capacity also when only lossy transmission of the noise is possible.
- In the MAC: The two users can *cooperate* in transmitting the noise (state) to the decoder.

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Summary

### MAC with SC SI

 $\mathcal{R}_{s-c}$  - the CH of all  $(R_1, R_2, \Gamma_1, \Gamma_2)$  satisfying

▶ Outline	$R_1$	$\leq$	$I(X_1; Y   X_2, U, V)$
Problem Formulation	$R_2$	$\leq$	$I(X_2; Y   X_1, U, V)$
Strictly Causal SI	$R_1 + R_2$	$\leq$	$I(X_1, X_2; Y   U, V)$
Single user and BC with	$B_1 \perp B_2$	<	$I(X_1, X_2, V; Y) - I(V; S)$
SC SI	$n_1 + n_2$	$\geq$	$I(X_1, X_2, V, I) = I(V, D)$
MAC with SC SI - main result	$\Gamma_{L}$	>	$E[\phi_k(X_k)],  k = 1, 2$
The coding scheme	⊥ <i>K</i>	<u>´</u>	$= [\varphi \kappa (1 \kappa)],  n = 1, 2$

#### for some joint distribution

Causal SI

Example

Independent SI streams

Summary

$$P_{U,V,X_1,X_2,S,Y} = P_S P_{X_1|U} P_{X_2|U} P_U P_{V|S} P_{Y|S,X_1,X_2}.$$

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 $X_1 - U - X_2$  $(X_1, U, X_2) \perp (V, S)$ 

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Summary

END

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**Theorem 1** (Strictly-Causal SI)  $\mathcal{R}_{s-c} \subseteq \mathcal{C}_{s-c}$ 

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V - S - Y

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The two codes are decoupled.

The total transmission time is divided into B + 1 blocks, each of length n.

Outline

**Problem Formulation** 

Strictly Causal SI

 Single user and BC with SC SI
 MAC with SC SI - main result

The coding scheme

Example

Causal SI

Independent SI streams

Summary

	The total transmission time is divided into $B + 1$ blocks, each of length $n$ .
▶Outline	First block - User 1 and User 2 transmit messages at rate $R_1$ and $R_2$ .
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Causal SI		
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Outline

SC SI

result

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The channel output at block b - 1 serves as the decoder's SI.

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Strictly Causal SI  Single user and BC with	The common mes
SC SI	
MAC with SC SI - main	previous block, b
result	•
<ul> <li>The coding scheme</li> <li>Example</li> </ul>	The channel outp
* Example	
Causal SI	The Wyner-Ziv co
Independent SI streams	

Summary

END

The total transmission time is divided into B + 1 blocks, each of length n.

First block - User 1 and User 2 transmit messages at rate  $R_1$  and  $R_2$ .

Block  $b \in [2:B]$ : the users cooperatively transmit a common message at rate  $R_0$ , and superimpose on it their private messages at rates  $R_1$ ,  $R_2$ .

The common message consists of a Wyner-Ziv codeword of the state in previous block, b - 1.

> The channel output at block b - 1 serves as the decoder's SI.

> The Wyner-Ziv codeword is independent of the state during its transmission.

# ine coung

The total transmission time is divided into B + 1 blocks, each of length n.

▶Outline	First block - User 1 and User 2 transmit messages at rate $R_1$ and $R_2$ .
Problem Formulation	Block $b \in [2:B]$ : the users cooperatively transmit a common message at rate
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Strictly Causal SI Single user and BC with	The common message consists of a Wyner-Ziv codeword of the state in
SC SI ▶MAC with SC SI - main result	previous block, $b-1$ .
<ul> <li>The coding scheme</li> <li>Example</li> </ul>	> The channel output at block $b - 1$ serves as the decoder's SI.
Causal SI	The Wyner-Ziv codeword is independent of the state during its transmission.
Independent SI streams	Block $B + 1$ : The users do not transmit private information. Transmit only the
Summary	common message, consisting of the Wyner-Ziv codeword of the state in block $B$ .

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**Backward decoding:** In block B + 1, the decoder decodes the state of block B, using the output of block B as side information.

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▶ The decoded state of block B is used to decode the messages sent at block B: private messages, and compressed state of block B − 1....

The Gaussian MAC where the state comprises the channel noise

Outline

**Problem Formulation** 

Strictly Causal SI

Single user and BC with SC SI
MAC with SC SI - main result
The coding scheme
Example

Causal SI

Independent SI streams

Summary

END

$$\begin{split} Y &= X_1 + X_2 + S, \qquad S \sim \mathcal{N}\left(0, \sigma_s^2\right) \\ \mathsf{E}\left[X_1^2\right] &\leq \Gamma_1, \qquad \mathsf{E}\left[X_2^2\right] \leq \Gamma_2. \end{split}$$

The Gaussian MAC where the state comprises the channel noise

Outline

**Problem Formulation** 

Strictly Causal SI

Single user and BC with SC SI MAC with SC SI - main result ▶ The coding scheme

Example

Causal SI

Independent SI streams

Summary

END

 $C_{s-c}(\Gamma_1,\Gamma_2)$  is the collection of all rate-pairs  $(R_1,R_2)$  satisfying

$$R_1 + R_2 \le \frac{1}{2} \log \left( 1 + \frac{(\Gamma_1^{\frac{1}{2}} + \Gamma_2^{\frac{1}{2}})^2}{\sigma_s^2} \right)$$

 $\mathsf{E}\left[X_1^2\right] \leq \Gamma_1, \qquad \mathsf{E}\left[X_2^2\right] \leq \Gamma_2.$ 

 $Y = X_1 + X_2 + S, \qquad S \sim \mathcal{N}\left(0, \sigma_s^2\right)$ 

Independent SI streams

SC SI

result

▶ The coding scheme

Example

Causal SI

**Problem Formulation** 

Strictly Causal SI Single user and BC with

MAC with SC SI - main

Outline

Summary

END



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The Gaussian MAC where the state comprises the channel noise

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I.e., the full cooperation line can be achieved.

Lapidoth & Steinberg, IZS 2010

The Gaussian MAC where the state comprises the channel noise

Example

Problem Formulation

Strictly Causal SI

```
Single user and BC with
SC SI
MAC with SC SI - main
result
The coding scheme
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Example

Causal SI

Independent SI streams

Summary

END

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Proof:

- p. 13/23

The Gaussian MAC where the state comprises the channel noise

**Problem Formulation** 

Strictly Causal SI

Single user and BC with SC SI
MAC with SC SI - main result
The coding scheme
Example

Causal SI

Independent SI streams

Summary

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Proof:

**Converse.** Since strictly causal SI does not increase the capacity of the single user channel, full cooperation is an upper bound.

## The Gaussian MAC where the state comprises the channel noise

Example

Outline

Problem Formulation

Strictly Causal SI

Single user and BC with SC SI
MAC with SC SI - main result
The coding scheme
Example

Causal SI

Independent SI streams

Summary

END

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Direct part. Two methods:

Causal SI

Example

Outline

SC SI

result

**Problem Formulation** 

MAC with SC SI - main

The coding scheme

Strictly Causal SI Single user and BC with

Independent SI streams

Summary

END

#### Example

The Gaussian MAC where the state comprises the channel noise

$$Y = X_1 + X_2 + S, \qquad S \sim \mathcal{N}\left(0, \sigma_s^2\right)$$
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Direct part. Two methods:

Good choice of random variables in our achievability region R<sub>s-c</sub>. (In some cases, R<sub>s-c</sub> is tight.)

\_\_\_\_\_

Proof:

END

Outline

SC SI

result

Example

Causal SI

Summary

**Problem Formulation** 

MAC with SC SI - main

The coding scheme

Independent SI streams

Strictly Causal SI Single user and BC with

#### Lapidoth & Steinberg, IZS 2010

The Gaussian MAC where the state comprises the channel noise

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Direct part. Two methods:

Example

- Good choice of random variables in our achievability region R<sub>s-c</sub>. (In some cases, R<sub>s-c</sub> is tight.)
- A Schalkwijk-Kailath algorithm

#### The region we had for the strictly causal case is still achievable

▶ Outline	$R_1$	$\leq$	$I(X_1; Y   X_2, U, V)$
Problem Formulation	$R_2$	$\leq$	$I(X_2; Y X_1, U, V)$
Strictly Causal SI	$R_1 + R_2$	$\leq$	$I(X_1, X_2; Y U, V)$
Causal SI	$R_1 + R_2$	$\leq$	$I(X_1, X_2, V; Y) - I(V; S)$
MAC with causal SI - main result	$\Gamma_k$	$\geq$	$E[\phi_k(X_k)],\qquad k=1,2$
Causal SI ▶MAC with causal SI - main	$R_1 + R_2$	$\leq$	$I(X_1, X_2, V; Y) - I(V$

#### with the Markov conditions

Independent SI streams

Summary

Example

END

 $X_1 - U - X_2$  $(X_1, U, X_2) \perp (V, S)$ V - S - Y

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<ul> <li>The naïve approach</li> <li>Example</li> </ul>	with the Markov conditions		

$$X_1 - U - X_2$$
$$(X_1, U, X_2) \perp (V, S)$$
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But now,  $X_1$ ,  $X_2$  can depend on S.

Independent SI streams

Summary

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The naïve approach			

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Independent SI streams

Summary

Example

END

$X_1 - U - X_2$
$(X_1, U, X_2) \perp (V, S)$
V - S - Y

But now,  $X_1$ ,  $X_2$  can depend on S.

 $\Rightarrow$  Use Shannon strategies on top of our block Markov scheme.

The region we had for the strictly causal case is still achievable

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Independent SI streams

Summary

END

 $X_1 - U - X_2$  $(X_1, U, X_2) \perp (V, S)$ V - S - Y

But now,  $X_1$ ,  $X_2$  can depend on S.

Replace  $(X_1, X_2)$  by  $(U_1, U_2)$  independent of S, and let

 $P_{X_1|U,U_1,S}, P_{X_2|U,U_2,S}$ 

 $\mathcal{R}_{cau}$  - the CH of all  $(R_1, R_2, \Gamma_1, \Gamma_2)$  satisfying

Outline

Problem Formulation

Strictly Causal SI

Causal SI

MAC with causal SI - main

result

The naïve approach

Example

# $$\begin{split} R_1 &\leq I(U_1; Y | U_2, U, V) \\ R_2 &\leq I(U_2; Y | U_1, U, V) \\ R_1 + R_2 &\leq I(U_1, U_2; Y | U, V) \\ R_1 + R_2 &\leq I(U_1, U, U_2, V; Y) - I(V; S) \\ \Gamma_k &\geq \mathsf{E}[\phi_k(X_k)], \qquad k = 1, 2 \end{split}$$

#### for some joint distribution

#### Independent SI streams

Summary

$$\begin{aligned} P_{U,U_1,U_2,V,X_1,X_2,S,Y} &= P_U P_{U_1|U} P_{U_2|U} P_{V|S} P_S \cdot \\ & P_{X_1|U,U_1,S} P_{X_2|U,U_2,S} P_{Y|S,X_1,X_2} \end{aligned}$$

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Problem Formulation

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#### Independent SI streams

Summary

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$$P_{U,U_1,U_2,V,X_1,X_2,S,Y} = P_U P_{U_1|U} P_{U_2|U} P_{V|S} P_S \cdot P_{X_1|U,U_1,S} P_{X_2|U,U_2,S} P_{Y|S,X_1,X_2}.$$

 $U_1 - U - U_2$ V - S - Y

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Outline

Problem Formulation

Strictly Causal SI

Causal SI

MAC with causal SI - main

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Independent SI streams

Summary

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 $U_1 - U - U_2$ V - S - Y $(U_1, U, U_2) \perp (V, S)$ 

 $\mathcal{R}_{cau}$  - the CH of all  $(R_1, R_2, \Gamma_1, \Gamma_2)$  satisfying

Outline

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Causal SI

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result

The naïve approach

Example

# $\begin{aligned} R_1 &\leq I(U_1; Y | U_2, U, V) \\ R_2 &\leq I(U_2; Y | U_1, U, V) \\ R_1 + R_2 &\leq I(U_1, U_2; Y | U, V) \\ R_1 + R_2 &\leq I(U_1, U, U_2, V; Y) - I(V; S) \\ \Gamma_k &\geq \mathsf{E}[\phi_k(X_k)], \qquad k = 1, 2 \end{aligned}$

#### for some joint distribution

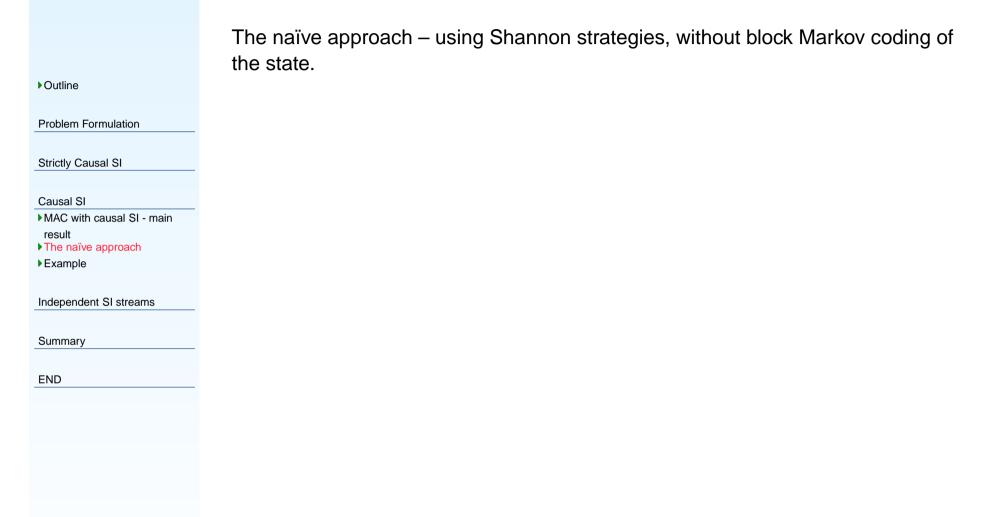
Independent SI streams

Summary

END

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Theorem 2 (Causal SI)  $\mathcal{R}_{cau} \subseteq \mathcal{C}_{cau}$ 



the state. It leads to the region of all  $(R_1, R_2)$  satisfying Outline  $R_1 < I(T_1; Y | T_2, Q)$ **Problem Formulation**  $R_2 \leq I(T_2; Y | T_1, Q)$ Strictly Causal SI  $R_1 + R_2 \leq I(T_1, T_2; Y|Q)$ Causal SI MAC with causal SI - main for some joint distribution  $P_Q P_{T_1|Q} P_{T_2|Q} P_{Y|T_1,T_2}$ . result The naïve approach Example Independent SI streams

Summary

END

The naïve approach – using Shannon strategies, without block Markov coding of

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all  $(R_1, R_2)$  satisfying

$R_1 \le I(T_1; Y   T_2, Q)$
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for some joint distribution  $P_Q P_{T_1|Q} P_{T_2|Q} P_{Y|T_1,T_2}$ . Here

 $T_k$ , k = 1, 2 are random Shannon strategies:

 $T_k \in \mathcal{T}_k$ , the set of mappings  $t_k : S \to \mathcal{X}_k$ 

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Problem Formulation

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Summary

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all  $(R_1, R_2)$  satisfying

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Q is a time sharing random variable,

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 $T_k \in \mathcal{T}_k$ , the set of mappings  $t_k : S \to \mathcal{X}_k$ 

Q is a time sharing random variable, and

$$P_{Y|T_1,T_2}(y|t_1,t_2) = \sum_{s \in \mathcal{S}} P_S(s) P_{Y|S,X_1,X_2}(y|s,t_1(s),t_2(s)).$$

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Independent SI streams

Summary

The naïve approach – using Shannon strategies, without block Markov coding of the state. It leads to the region of all  $(R_1, R_2)$  satisfying Outline  $R_1 \leq I(T_1; Y | T_2, Q)$ **Problem Formulation**  $R_2 \leq I(T_2; Y | T_1, Q)$ Strictly Causal SI  $R_1 + R_2 \leq I(T_1, T_2; Y|Q)$ Causal SI MAC with causal SI - main for some joint distribution  $P_Q P_{T_1|Q} P_{T_2|Q} P_{Y|T_1,T_2}$ . result The naïve approach Example We denote this region as  $\mathcal{R}^{naïve}$ . Independent SI streams Summary END

-  $\mathcal{R}_{cau}$  contains the region of the naïve approach, since we can always choose degenerate *V*.

Problem Formulation

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Causal SI

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END

- In some cases, the inclusion is strict.

The noiseless binary MAC with input selector:

Outline

**Problem Formulation** 

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 The naïve approach

Example

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Summary

END

 $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y} = \{0, 1\}, \quad \mathcal{S} = \{1, 2\}, \quad P_S(S = 2) = p > 0.5$ 

The noiseless binary MAC with input selector:

Outline

Problem Formulation

Strictly Causal SI

 $Y = X_S$ 

 $\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y} = \{0, 1\}, \quad \mathcal{S} = \{1, 2\}, \quad P_S(S = 2) = p > 0.5$ 

Causal SI

 MAC with causal SI - main result
 The naïve approach

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END

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Causal SI

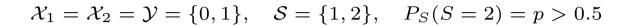
 MAC with causal SI - main result
 The naïve approach

Example

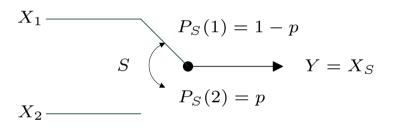
Independent SI streams

Summary

END



 $Y = X_S$ 



The noiseless binary MAC with input selector:

Outline

Problem Formulation

Strictly Causal SI

Causal SI

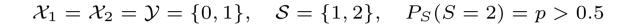
 MAC with causal SI - main result
 The naïve approach

Example

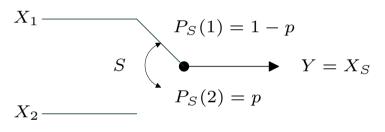
Independent SI streams

Summary

END



 $Y = X_S$ 



- If the decoder knows S, user 1 can transmit at rate 1 - p.

- Hence, 1 - p is an upper bound on the transmission rate of user 1 in our model.

Problem Formulation

Strictly Causal SI

Causal SI

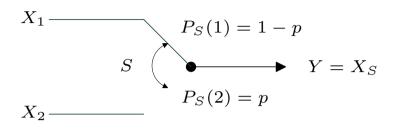
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Outline

**Problem Formulation** 

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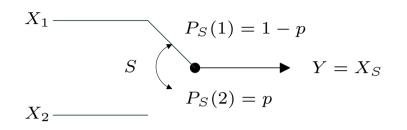
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With a proper choice of random variables in  $\mathcal{R}_{cau}$ 

$$(R_1, R_2) = \left(\min\{1 - p, 1 - H_b(p)\}, 0\right) \in \mathcal{R}_{cau}$$

(Observe – achieves the maximal rate of user 1 for  $p \ge H_b(p)$ .)



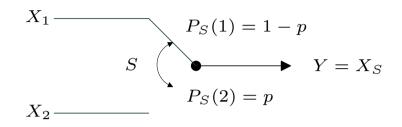
**Problem Formulation** 

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$$(R_1, R_2) = \left(\min\{1 - p, 1 - H_b(p)\}, 0\right) \in \mathcal{R}_{\mathsf{cau}}$$

Independent SI streams

Summary

END

(Observe – achieves the maximal rate of user 1 for 
$$p \ge H_b(p)$$
.)

The maximal rate of user 1 in the naïve approach is

$$R_{2,\max}^{\text{naïve}} = \log_2\left(1 + (1-p)p^{rac{p}{1-p}}
ight)$$
 bits



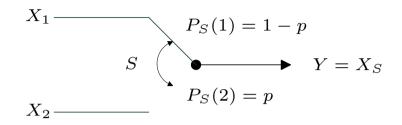
**Problem Formulation** 

Strictly Causal SI

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MAC with causal SI - main result
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Example



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Independent SI streams

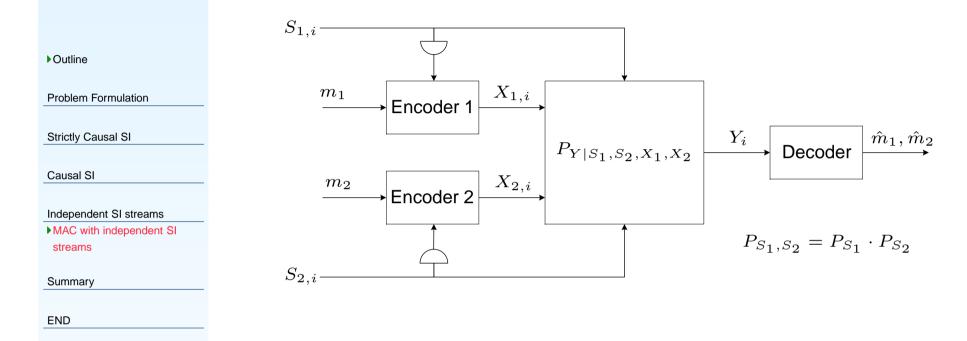
Summary

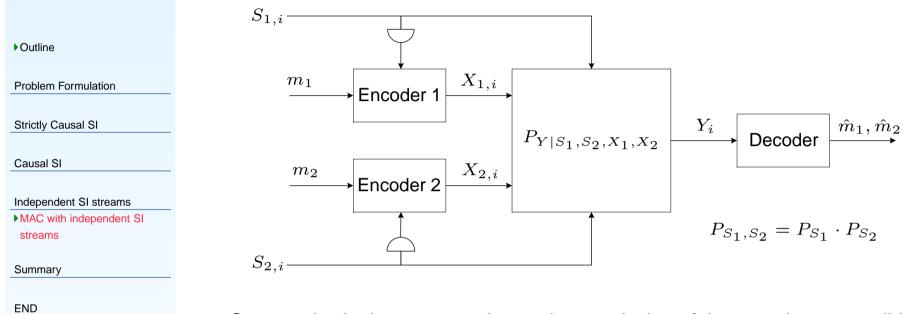
END

$$R_{2,\max}^{\text{naïve}} = \log_2\left(1 + (1-p)p^{\frac{p}{1-p}}\right) \quad \text{bits}$$

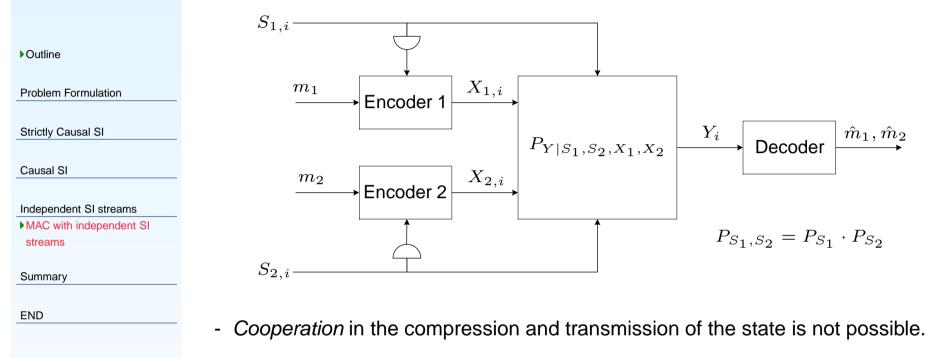
For sufficiently large value of p,

 $R_{2,\max}^{\text{naïve}} < \min \left\{1-p, 1-H_b(p)\right\}$ 

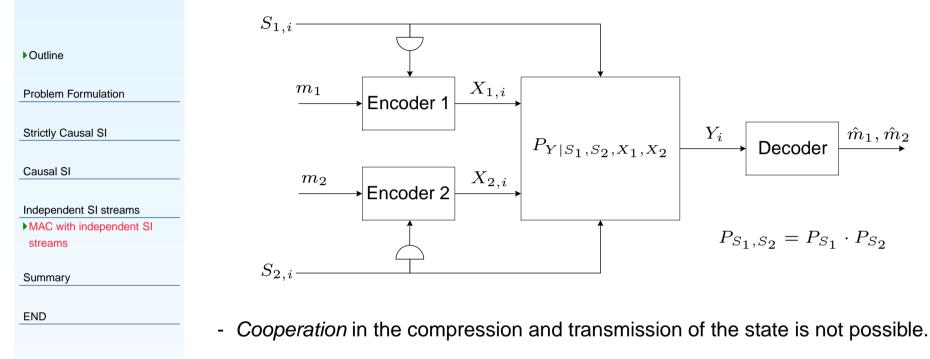




- Cooperation in the compression and transmission of the state is not possible.



- Yet, compression and transmission of the states to the decoder is beneficial, and enlarges the capacity region of the MAC.



- Yet, compression and transmission of the states to the decoder is beneficial, and enlarges the capacity region of the MAC.
- Utilize distributed Wyner-Ziv compression and block Markov coding (ISIT 2010).

# Summary

Derived achievable region for the MAC with common strictly causal SI, based on block Markov encoding of the state.

- Strictly causal SI enlarges the capacity region of the MAC.
- Extended the results to causal SI
- The new region for causal Si is strictly better that the region obtained by the naïve approach.
- Strictly causal SI is beneficial even when the states available at the encoders are independent (ISIT 2010).

Outline

Problem Formulation

Strictly Causal SI

Causal SI

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END

# Thank You!