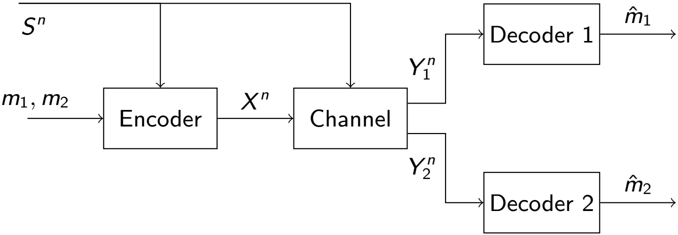
The Degraded Broadcast Channel with Non-Causal Action-Dependent Side Information

Yossef Steinberg

ISIT 2013

Problem formulation

The "regular" state-dependent BC:



- Channel encoder:

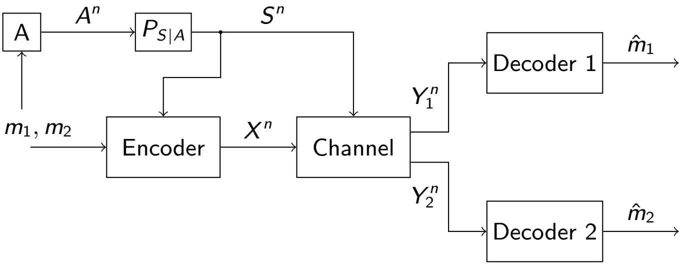
$$X_i = f(m_1, m_2, S^n) \quad \text{(non-causal SI)}$$
$$X_i = f(m_1, m_2, S^i) \quad \text{(causal SI)}$$

- $\frac{1}{n}\sum_{i=1}^{n} \Lambda(X_i) \le \lambda$

$$P\left((\hat{m}_1, \hat{m}_2) \neq (m_1, m_2)\right) \le \epsilon$$

Problem formulation

Action-dependent states:



Encoding is performed in two parts:

- Given the pair of messages, an *action sequence* A^n is created.
  The actions generate a sequence of states S^n, via $P_{S|A}$.
  S^n is available at the encoder (causally or noncausally).
- The encoder produces the channel input as a function of the messages and the states S^n.

Problem formulation

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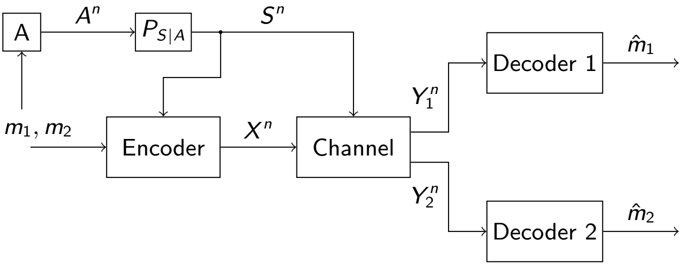

- Two possible models

$$X_i = f(m_1, m_2, S^n) \quad \text{(non-causal SI)}$$
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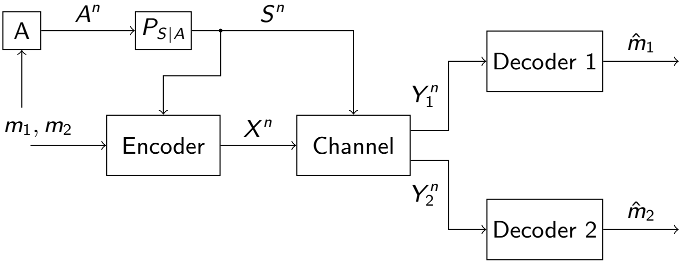

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$$X_i = f(m_1, m_2, S^n) \quad \text{(non-causal SI)}$$
$$X_i = f(m_1, m_2, S^i) \quad \text{(causal SI)}$$

- Causal case solved [S & Weissman 2012], [Ahmedi & Simeone 2012].

Motivation

- Controlling the channel: sometimes, the user can affect the channel statistics (state), albeit at a certain cost.

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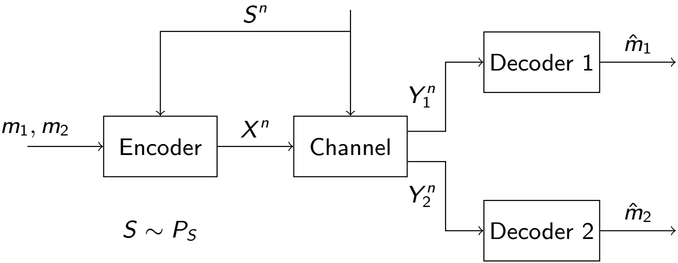
Motivation

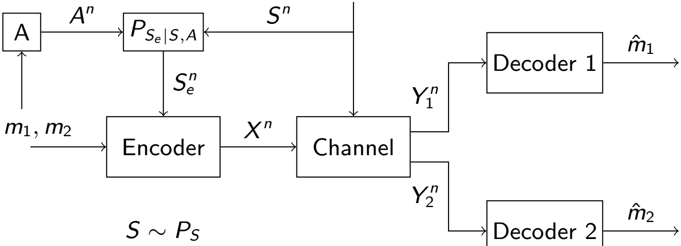
Cost of retrieving SI:



In "regular" channel coding with SI, state is produced by nature (not by actions). It is either *available* at the encoder, or *absent*. No intermediate situation, and no cost on retrieving it.

Motivation

Cost of retrieving SI:



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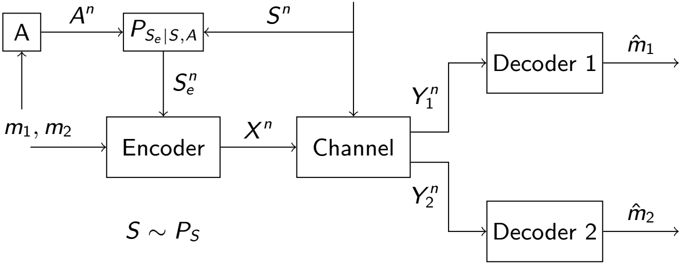

- S is produced by nature

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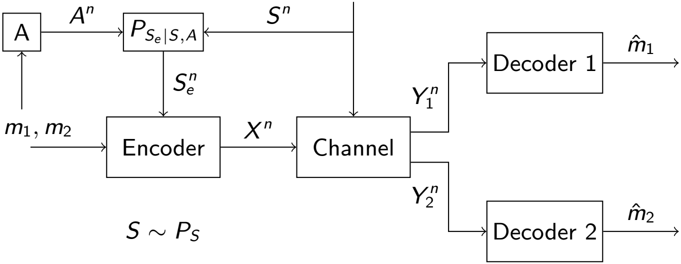

- S is produced by nature
- Side information is not available for free - we have to "go out and get it," or install expensive (and noisy) sensors to get it.

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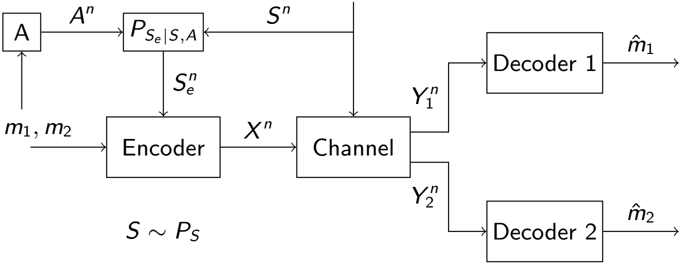
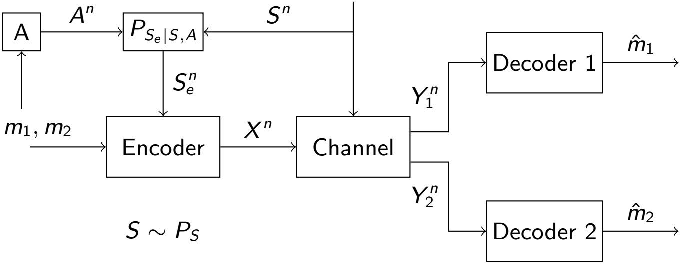


- S is produced by nature
- Side information is not available for free - we have to "go out and get it," or install expensive (and noisy) sensors to get it.
- The actions determine the availability (and quality) of side information at the encoder - S_e.

Motivation

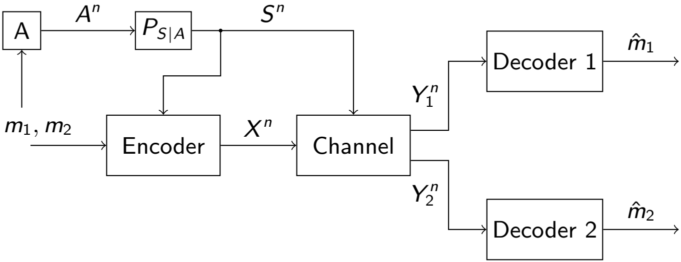
Cost of retrieving SI:



Probing capacity. Introduced in the context of single user channels by Asnani, Permuter, & Weissman, 2010.

Problem formulation

The basic setup:



- Memoryless channel
- Non causal SI: $X_i = f(m_1, m_2, S^n)$
- Cost on input and actions:

$$\frac{1}{n}\sum_{i=1}^{n}\Lambda(A_i, X_i) \leq \lambda$$

Previous results

Action dependent channels and sources

- Weissman 2010 - Introduced action dependent channels.
  - Capacity of single user channels, causal and non-causal models.
  - Bounds on the capacity of rewrite channels.
  - Connection to certain MAC models.
- H. Asnani, H. Permuter, & T. Weissman 2010 (arXiv) - Probing capacity: to observe or not to observe the side information? ($P_{S_e|S,A}$).
- Permuter & Weissman 2011 - Actions in the context of source coding: the side information vending machine
- Y.-K. Chia, H. Asnani, & T. Weissman 2011 (arXiv) - Multiterminal source coding with action dependent side information

Previous results

Action dependent single user channels

- Causal case (Weissman 2010):

$$C_c = \max I(U, A; Y)$$
$$\mathrm{E}\left[\Lambda(A, X)\right] \leq \lambda$$

$$P_{U,A} P_{S|A} P_{X|S,U,A} P_{Y|S,X}$$

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$$C_c = \max I(U, A; Y) \quad = I(A; Y) + I(U; Y|A)$$
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$$E[\Lambda(A, X)] \leq \lambda$$

$$P_{U,A} P_{S|A} P_{X|S,U,A} P_{Y|S,X}$$

- Non causal case (Weisman 2010)

$$C_{nc} = \max I(U, A; Y) - I(U; S|A)$$
$$E[\Lambda(A, X)] \leq \lambda$$

$$P_A P_{S|A} P_{U|S,A} P_{X|S,U,A} P_{Y|S,X}$$

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In both cases, X can be taken to be a deterministic function of (U,S), and A a deterministic function of U.

Previous results

State dependent broadcast channels

- S 2002, 2005 - Degraded, state dependent BC:
  - Capacity region for causal SI
  - Inner and outer bounds for non-causal SI
  - Capacity region for non-causal SI, where the stronger user is informed

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  - Capacity region for causal SI
  - Inner and outer bounds for non-causal SI
  - Capacity region for non-causal SI, where the stronger user is informed
- S & Shamai ISIT 2005:
  - Inner bounds for the general state dependent BC (Marton region + GP).

Previous results

State dependent BC

A state dependent BC $P_{Y_1,Y_2|S,X}$ is called physically degraded if

$$P_{Y_1,Y_2|S,X} = P_{Y_1|S,X} \cdot P_{Y_2|Y_1}$$

and stochastically degraded if

$$P_{Y_2|S,X}(y_2|s,x) = \sum_{y_1} P_{Y_1,Y_2|S,X}(y_1,y_2|s,x) \cdot W_{Y_2|Y_1}(y_2|y_2)$$

for *some* $W_{Y_2|Y_1}$.

Previous results

BC + Actions, the causal case

$\mathcal{R}_c$ - the collection of all (λ, R_1, R_2) such that

$$R_1 \leq I(U, A; Y_1|K)$$
$$R_2 \leq I(K; Y_2)$$
$$\mathsf{E}\left[\Lambda_k(A, X)\right] \leq \lambda_k, \quad k = 1, 2, \ldots, d$$

for some

$$P_{A,K,U,S,X,Y,Z} = P_{K,U} P_{A|K,U} P_{X|A,K,U,S} P_{S|A} P_{Y_1,Y_2|S,X}.$$

Theorem

For the degraded BC with action dependent states and causal SI

$$\mathcal{C}_c = \mathcal{R}_c.$$

[S. & Weissman, 2012], [Ahmedi & Simeone, 2012].

Previous results

BC + Actions, the causal case

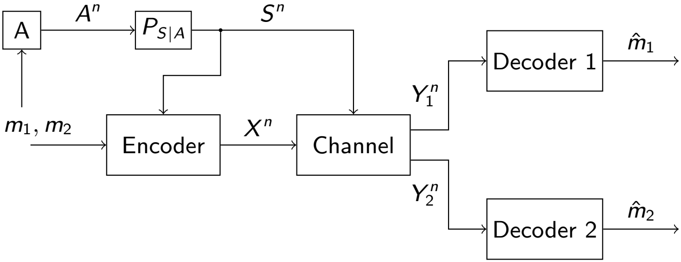
$$R_1 \leq I(U, A; Y_1|K) \quad = I(A; Y_1|K) + I(U; Y_1|K, A)$$

$$R_2 \leq I(K; Y_2)$$

$$\mathsf{E}\left[\Lambda_k(A, X)\right] \leq \lambda_k, \quad k = 1, 2, \ldots, d$$

$$P_{A,K,U,S,X,Y,Z} = P_{K,U} P_{A|K,U} P_{X|A,K,U,S} P_{S|A} P_{Y_1,Y_2|S,X}.$$

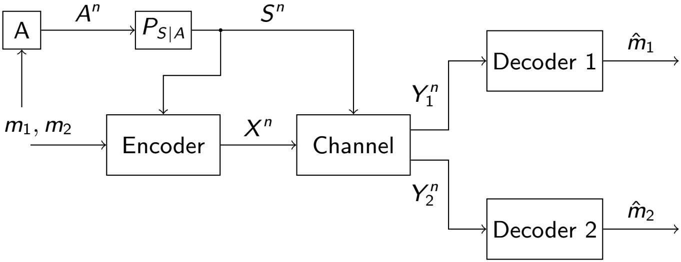
Main results



- Memoryless channel
- Non causal SI: $X_i = f(m_1, m_2, S^n)$
- Cost on input and actions:

$$\frac{1}{n}\sum_{i=1}^{n} \Lambda(A_i, X_i) \leq \lambda \qquad (\Lambda,\ \lambda \in R^d)$$

Main results



- Capacity region: $\mathcal{C}_{\text{nc}}$
- $\mathcal{C}_{\text{nc}}$ depends on $P_{Y_1,Y_2|S,X}$ only via $P_{Y_1|S,X}$ and $P_{Y_2|S,X}$.

$\Rightarrow$ No distinction has to be made between physically and stochastically degraded channels. General term: *degraded*.

Main results

Inner bound

$\mathcal{R}_{\mathrm{i}}$ - the collection of all (λ, R_1, R_2) such that

$$R_2 \leq I(K, A_2; Y_2) - I(K; A, S|A_2)$$

$$R_1 \leq I(U, A; Y_1|K, A_2) - I(U; S|K, A_2, A)$$
$$\mathsf{E}\left[\Lambda_k(A, X)\right] \leq \lambda_k, \quad k = 1, 2, \ldots, d$$

for some

$$P_{A,A_2,K,U,S,X,Y_1,Y_2} = P_{A,A_2} P_{S|A} P_{K,U,X|A_2,A,S} P_{Y_1,Y_2|S,X}.$$

Theorem

For the degraded BC with action dependent states and causal SI

$$\mathcal{R}_{\mathrm{i}} \subseteq \mathcal{C}_{\mathrm{nc}}.$$

Main results

Inner bound

$\mathcal{R}_i$ - the collection of all (λ, R_1, R_2) such that

$$
\begin{aligned}
R_2 &\le I(K, A_2; Y_2) - I(K; A, S|A_2) \\
&= I(A_2; Y_2) + I(K; Y_2|A_2) - I(K; A, S|A_2) \\
R_1 &\le I(U, A; Y_1|K, A_2) - I(U; S|K, A_2, A) \\
&\mathrm{E}\left[\Lambda_k(A, X)\right] \le \lambda_k, \quad k = 1, 2, \ldots, d
\end{aligned}
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for some

$$P_{A,A_2,K,U,S,X,Y_1,Y_2} = P_{A,A_2} P_{S|A} P_{K,U,X|A_2,A,S} P_{Y_1,Y_2|S,X}.$$

Theorem

For the degraded BC with action dependent states and causal SI

$$\mathcal{R}_i \subseteq \mathcal{C}_{nc}.$$

Main results

Properties of $\mathcal{R}_{\mathrm{i}}$

$$
\begin{aligned}
R_1 &\le I(U, A; Y_1|K, A_2) - I(U; S|K, A, A_2) \\
R_2 &\le I(K, A_2; Y_2) - I(K; A, S|A_2) \\
\mathsf{E}\left[\Lambda_k(A, X)\right] &\le \lambda_k, \quad k = 1, 2, \ldots, d
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$$

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P_{A,A_2,K,U,S,X,Y_1,Y_2} = {} & P_{A_2,K,U} P_{A|A_2,K,U} P_{X|A,A_2,K,U,S} \\
& \cdot P_{S|A,A_2,K,U} P_{Y_1,Y_2|S,X} \cdot
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Main results

Properties of $\mathcal{R}_i$

$$\begin{aligned} R_1 &\le I(U,A;Y_1|K,A_2) - I(U;S|K,A,A_2) \\ R_2 &\le I(K,A_2;Y_2) - I(K;A,S|A_2) \\ \mathrm{E}\left[\Lambda_k(A,X)\right] &\le \lambda_k, \quad k = 1,2,\ldots,d \end{aligned}$$

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- $\mathcal{R}_i$ is convex.

Main results

Properties of $\mathcal{R}_\text{i}$

$$\begin{aligned} R_1 &\le I(U, A; Y_1|K, A_2) - I(U; S|K, A, A_2) \\ R_2 &\le I(K, A_2; Y_2) - I(K; A, S|A_2) \\ \mathsf{E}\left[\Lambda_k(A, X)\right] &\le \lambda_k, \quad k = 1, 2, \ldots, d \end{aligned}$$

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- $\mathcal{R}_\text{i}$ is convex.
- To exhaust $\mathcal{R}_\text{i}$, $P_{A|A_2,K,U}$ and $P_{X|A,A_2,K,U,S}$ can be $0-1$ laws.

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Properties of $\mathcal{R}_i$

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- $\mathcal{R}_i$ is convex.
- To exhaust $\mathcal{R}_i$, $P_{A|A_2,K,U}$ and $P_{X|A,A_2,K,U,S}$ can be $0-1$ laws. Can drop the A from the bound on R_1.

Main results

Properties of $\mathcal{R}_i$

- Bounds on alphabets

$$
\begin{aligned}
|\mathcal{A}_2| &\leq |\mathcal{ASX}| + 1 \\
|\mathcal{K}| &\leq |\mathcal{ASX}| \left(|\mathcal{ASX}| + 1\right) + 1 \\
|\mathcal{U}| &\leq |\mathcal{ASX}| \left[|\mathcal{ASX}| \left(|\mathcal{ASX}| + 1\right) + 1\right] \\
&\quad \cdot \left[|\mathcal{ASX}| + 1\right]
\end{aligned}
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Main results

Proof technique

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- BC: In the problem formulation, the action depends on both messages, m_1 and m_2.
  - Cannot start with $A^n(m_1, m_2)$. (The signal for the weaker user, K, is conditioned on it.)
  - *Some* action should be there.

Main results

Proof technique

$$R_2 \leq I(K, A_2; Y_2) - I(K; A, S|A_2)$$
$$R_1 \leq I(U, A; Y_1|K, A_2) - I(U; S|K, A_2, A)$$

$$P_{A,A_2,K,U,S,X,Y_1,Y_2} = P_{A,A_2} P_{S|A} P_{K,U,X|A_2,A,S} P_{Y_1,Y_2|S,X}.$$

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$$P_{A,A_2,K,U,S,X,Y_1,Y_2} = P_{A,A_2} P_{S|A} P_{K,U,X|A_2,A,S} P_{Y_1,Y_2|S,X}.$$

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- Generate actions $A^n(m_1, m_2)$ by $\prod_{i=1}^n P_{A|A_2}(\cdot|A_{2,i}(m_1))$
- Generate a codebook $K^n(j, m_2)$ by $\prod_{i=1}^n P_{K|A_2}(\cdot|A_{2,i}(m_2))$
- Binning 2: j_{m_2} is the smallest integer s.t.

$$(K^n(j, m_2), A_2^n(m_2), A^n(m_1, m_2), s^n) \in \mathcal{T}$$

Main results

Proof technique

$$R_2 \leq I(K, A_2; Y_2) - I(K; A, S|A_2)$$
$$R_1 \leq I(U, A; Y_1|K, A_2) - I(U; S|K, A_2, A)$$

$$P_{A,A_2,K,U,S,X,Y_1,Y_2} = P_{A,A_2} P_{S|A} P_{K,U,X|A_2,A,S} P_{Y_1,Y_2|S,X}.$$

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Main results

Outer bound

$\mathcal{R}_o$ - all (R_1, R_2, λ) such that

$$\begin{aligned} R_2 &\leq I(K, A_2; Y_2) - I(K; A, S|A_2) \\ R_1 &\leq I(U, A; Y_1|K) - I(U; S|K, A_2, A) \\ R_1 + R_2 &\leq I(U, K, A; Y_1) - I(U, K; S|A) \\ \mathsf{E}\left[\Lambda_k(A, X)\right] &\leq \lambda_k, \quad k = 1, \ldots, d \end{aligned}$$

for some $P_{A,A_2,K,U,S,X,Y_1,Y_2} \in \mathcal{P}$.

Main results

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for some $P_{A,A_2,K,U,S,X,Y_1,Y_2} \in \mathcal{P}$.

Theorem

For any degraded BC with action-dependent non-causal SI

$$\mathcal{C}_{nc} \subseteq \mathcal{R}_o$$

Main results

Properties of $\mathcal{R}_o$

$\mathcal{R}_o$ - all (R_1, R_2, λ) such that

$$\begin{aligned} R_2 &\leq I(K, A_2; Y_2) - I(K; A, S|A_2) \\ R_1 &\leq I(U, A; Y_1|K) - I(U; S|K, A_2, A) \\ R_1 + R_2 &\leq I(U, K, A; Y_1) - I(U, K; S|A) \\ \mathrm{E}\left[\Lambda_k(A, X)\right] &\leq \lambda_k, \quad k = 1, \ldots, d \end{aligned}$$

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Main results

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for some $P_{A,A_2,K,U,S,X,Y_1,Y_2} \in \mathcal{P}$.

- Convex

Main results

Properties of $\mathcal{R}_o$

$\mathcal{R}_o$ - all (R_1, R_2, λ) such that

$$
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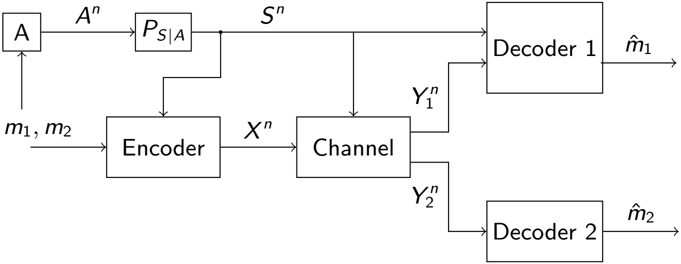
R_1 &\le I(U, A; Y_1|K) - I(U; S|K, A_2, A) \\
R_1 + R_2 &\le I(U, K, A; Y_1) - I(U, K; S|A) \\
\mathsf{E}\left[\Lambda_k(A, X)\right] &\le \lambda_k, \quad k = 1, \ldots, d
\end{aligned}
$$

for some $P_{A,A_2,K,U,S,X,Y_1,Y_2} \in \mathcal{P}$.

- Convex
- Bounds on alphabets

Main results

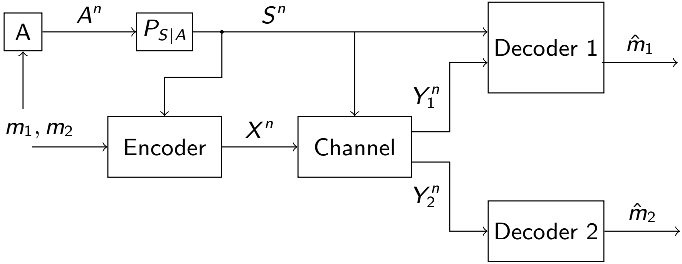
Informed stronger decoder



- Even without actions, the state-dependent degraded BC with non-causal SI is still an open problem.
- Solved for the case where the stronger user is informed.

Main results

Informed stronger decoder



- Even without actions, the state-dependent degraded BC with non-causal SI is still an open problem.
- Solved for the case where the stronger user is informed.
- For the action-dependent case, we need to restrict the class of costs $\Lambda(A, X)$.

Main results

Informed stronger user

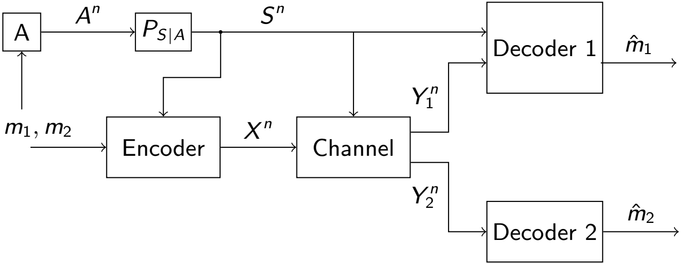
- *Separated cost functions* Λ^{sep}:
  Each of the components of Λ depends either only on the actions or only on the channel input:

$$\Lambda^{\text{sep}}_{k'}(A^n, X^n) = \Lambda^{\text{sep}}_{k'}(A^n), \quad 1 \le k' \le d',$$
$$\Lambda^{\text{sep}}_{k}(A^n, X^n) = \Lambda^{\text{sep}}_{k}(X^n), \quad d'+1 \le k \le d,$$

for some $0 \le d' \le d$.

Main results

Informed stronge user



$\mathcal{R}_{\text{nc}}$ - all (R_1, R_2, λ) such that

$$R_2 \leq I(K, A_2; Y_2) - I(K; S|A_2)$$
$$R_1 \leq I(A; S|A_2) + I(X; Y_1|S, K, A_2)$$
$$\mathrm{E}\left[\Lambda_k^{\text{sep}}(A, X)\right] \leq \lambda_k, \quad k = 1, 2, \ldots, d$$

for some

$$P_{A,A_2} P_{S|A} P_{K|A,A_2,S} P_{X|K,A_2,S} P_{Y_1,Y_2|S,X}$$

Main results

Informed stronger user

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$$

$$
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$$

Theorem

For any DBC with action-dependent non-causal SI, informed stronger user, and separated cost functions

$$
\mathcal{C}_{\mathrm{nc}} = \mathcal{R}_{\mathrm{nc}}
$$

Main results

Informed stronger user

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Main results

Informed stronger user

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$$P_{A,A_2} P_{S|A} P_{K|A,A_2,S} P_{X|K,A_2,S} P_{Y_1,Y_2|S,X}$$

- User 2: As in single user channel, with actions A_2.

Main results

Informed stronger user

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- User 2: As in single user channel, with actions A_2.
- User 1: coding in two *separate* stages:
  - Via the actions A directly to S
  - Via X to Y_1, conditioned on (S, K, A_2).

Main results

Informed stronger user

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  - Via the actions A directly to S
  - Via X to Y_1, conditioned on (S, K, A_2).
- Conditioned on (S, K, A_2), X indep of A.

Main results

Informed stronger user

Converse:

Main results

Informed stronger user

Converse:

- User 2 - as in single user.

Main results

Informed stronger user

Converse:

- User 2 - as in single user.
- User 1 - can get a bound of the form

$$nR_1 - n\epsilon_n \leq \sum_{i=1}^{n} I(A_i; S_i|A_{2,i}) + I(X_i; Y_{1,i}|S_i, K_i, A_{2,i})$$

Main results

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$$\Longrightarrow \Lambda^{\text{sep}}$$

Summary

- Developed inner and outer bounds on the capacity region of the degraded BC with action-dependent states and non-causal SI.
- The case of informed stronger user is solved.
- Future work: General (non-informed) setting. Good examples.